

Novel Geometry and Symmetry in String Theory

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Colloquium at NTHU, 05/12/2012

Outline

- 1 Basic String Theory
- 2 Novel Geometry and Symmetry in String Theory
 - Duality Symmetries and M-Theory
 - Noncommutative Geometry
 - Dynamical Generation of Spacetime
 - Holographic Principle and AdS/CFT Correspondence
 - Generalization of Yang-Mills Gauge Symmetry from M5-Branes
- 3 Conclusions

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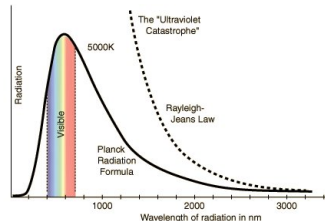
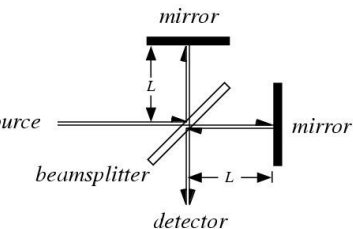
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Two Clouds of Theoretical Physics in 1900



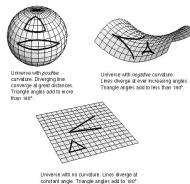
Two Clouds of Theoretical Physics

In 1900, in a lecture titled "Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light, Lord Kelvin, an influential British physicist, famously proclaimed that physics was over, except for two small clouds on the horizon.

These "clouds" turned out to be the clues that led to the discovery of quantum mechanics and special relativity.

Synthesis and Unification

i. The generalization of special relativity leads to general relativity



Combination of space and time into spacetime naturally calls for curvature

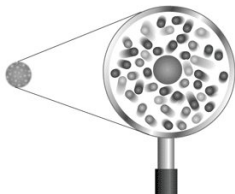
ii. The combination of quantum mechanics and special relativity leads to QFT.

Resolving the conflict of causality in special relativity and quantum mechanics forces on us a new structure: the existence of anti-particles.

Another Big Cloud

QFT and general relativity describes the world very well. However, the two are not compatible. For example,

- There exists infinities in the quantum effects of gravity that we do not know how to “handle” (Non-renormalizability).



- Physical information is lost in a blackhole (blackhole information paradox)



Quantization of gravity is one of the most important problem of theoretical physics.

There is no reason not to believe that this “cloud” will lead us to a deeper understanding of the Nature.

Basics of String Theory

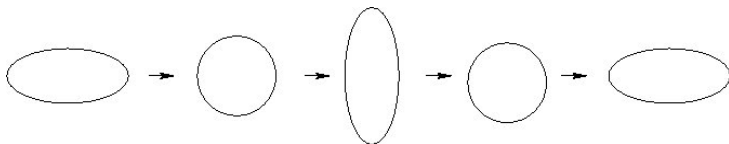
String theory is a theory still under construction. It is almost 40 years old.

String theory is a reasonable modification of QFT:

QFT = special relativity + quantum mechanics + point-like particles

String Theory = special relativity + quantum mechanics + **strings**

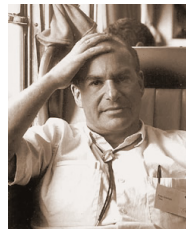
- In addition to translation, string is also capable of **vibration**
There is a unique fundamental object (i.e. string). **Different vibrational mode** of the elementary string give rises to particle excitation that has **mass and spin** depending on the vibrational mode.



String Unification

One of the string excitations is a spin two massless particle and has the right properties of the graviton.

Scherk and Schwarz proposed in 1974 to use string theory to quantize gravity and to unify all forces.



String Theory unifies matters and interactions!!

First string revolution (1984)

The original bosonic string theory suffered from a number of problems: It has no fermions. It lives in 26 dimensions and it has a tachyon in its spectrum.

These problems were resolved with the introduction of two crucial concepts:

- i. **Supersymmetry**: supersymmetry introduces fermions and removes the tachyon.
- ii. **Compactification**: superstring lives in 10 dimensions. The extra 6 spatial dimensions can curl up and become very small in a gravity theory. Most importantly this is a dynamical process in string theory and do not needed to be put in by hand.

As a result, five consistent superstring theories were found:

Type I, Type IIA, Type IIB, HE, HO

(HE, HO: Heterotic string with $E_8 \times E_8$ and $SO(32)$ gauge group)

These string theories live in 10 dimensional spacetime and has only two parameters:

mass parameter α' (string tension);
string coupling constant g .

- Each of these superstring theory respects different supersymmetries and gauge symmetries.

However there are at least two unsatisfactory aspects:

Q1. Uniqueness of String Theory

Why so many? Which is the ONE to explain our world?

Q2. Uniqueness of the Vacuum

There are many ways one can compactify the extra dimensions. Why is a particular one chosen?

Some of these questions got answers in the last decade due to a number of advances in our understanding of the geometry and symmetry of string theory.

Suprisingly, it's realized now:

String theory is not just a theory of strings!

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T-duality and Mirror Symmetry

- It was found that different geometries for the extra dimensions can be physically equivalent to each other. This is called the **target space duality** (T-duality). For example, string theory A with large compact dimensions is equivalent to another string theory B with a small compact dimensions:

$$R_A = \frac{l_s^2}{R_B}.$$

- There are two such relations

$$\text{IIA} \xleftrightarrow{T} \text{IIB},$$

$$\text{HE} \xleftrightarrow{T} \text{HO}.$$

- This should not be too surprising as it can be expected that string probes geometry in a very different way from a particle!
- Because of this novel property of string dynamics, string theory implies a generalized uncertainty principle

$$\Delta X \geq \frac{1}{\Delta P} + \alpha_s^2 \Delta P$$

One can expect even more surprising and richer geometrical features, as well as novel physical consequences in string theory!

S duality

- It was also found that some string theory with a coupling g is equivalent to another one with a coupling $g' = 1/g$. This is called the strong-weak duality (S duality).
- Two examples are:

$$\begin{array}{ccc} \text{I} & \overset{S}{\longleftrightarrow} & \text{HO} \\ \text{IIB} & \overset{S}{\longleftrightarrow} & \text{IIB} \end{array}$$

.

Eleven dimensions and M-theory

- Q. What happens to the other two string theories (IIA and HE) when g grows large?
- It turns out that the strongly coupled string theory is better described in terms of an eleven dimensional theory. A new dimension is grown with size:

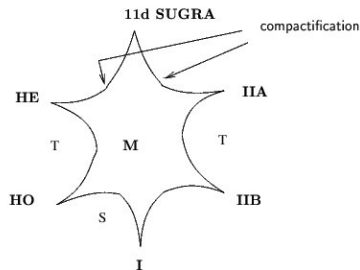
size of 11th dimension $R_{11} = gL$.

- This new dimension is a circle in the IIA case and an interval in the HE case:

$\lim_{g \rightarrow \infty} \text{IIA} = \text{M theory on a circle,}$

$\lim_{g \rightarrow \infty} \text{HE} = \text{M theory on an interval.}$

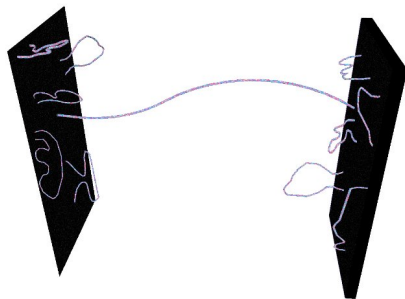
The five superstrings are considered to be different limits of the 11-dimensional M-theory.



The underlied unifying theory is called the **M-theory** (Mother, “W”itten, or ...).

D-branes

D-branes play very important roles in the establishment of this web of duality.



A Dp -brane is an extended object (with p spatial dimensions and one time) upon which open strings can end on them. They are solitonic, somewhat like domain wall or vortex string in a gauge theory.

Matrix Theory

BFSS matrix model

- Bank-Fishler-Shenker-Susskind (BFSS) proposed that M-theory can be described in terms of the maximally supersymmetric $U(N)$ quantum mechanics:

$$S = \int dt (D_t X^I)^2 - [X^I, X^J]^2 + \text{fermions}$$

Here the variables X^I are $N \times N$ unitary matrix, $I = 1, \dots, 9$, and denotes the space coordinates as seen by N D0-branes (D-particles).

M is for Matrix!

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- The studies of noncommutative geometry has led to a deeper understanding of certain properties of spacetime at the Planck scale.
- It also has nontrivial signature in the particle physics sector.

- It was discovered recently that a similar phenomena occurs for the M5-brane in the presence of a constant 3-form C -field background

$$[X^i, X^j, X^k] = i\theta^{ijk}$$

Chu, Sembi (2010)

$$[f, g, h] := fgh + ghf + hfg - fhg - gfh - hgf,$$

is the quantum Nambu bracket (Nambu 1973) defined on ordinary operators.



This is called the **Quantum Nambu Geometry**.

The construction of QFT on Quantum Nambu geometry is potentially a very important topic both theoretically and phenomenologically.

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A very fundamental question (perhaps crazy) of theoretical physics is about the nature of spacetime. What exactly is spacetime? Why is spacetime 4-dimensional? Why is there a time? Is spacetime a fundamental or a derived concept?

Quotes:

I am almost certain that space and time are illusions (Nathan Seiberg).

The real change that's around the corner [is] in the way we think about space and time. We haven't come to grips with what Einstein taught us. But that's coming. And that will make the world around us seem much stranger than any of us can imagine. (David Gross)

Spacetime generation

- In string theory, or its eleventh dimensions completion M-theory, allowable spacetime is determined dynamically as solution to the equation of motion.
- EOM of the BFSS theory is:

$$D_t^2 X^I - [X^J, [X^I, X^J]] = 0$$

- For example, it has the time independent soln

$$[X^i, X^j] = i\theta^{ij}\mathbf{1}, \quad \theta^{ij} = \text{constant}$$

This vacuum solution describes a noncommutative spacetime. Other solutions are possible.

- In this example, spacetime and its properties (dimensions and existence of noncommutativity) is obtained dynamically!
- There has also been some analysis based on Monte Carlo suggesting that 4 dimensional spacetime is energetically favorable over other dimensionality.
- This emergence of spacetime from a matrix model is very much in line with the philosophy that spacetime is a derived concept.

The world is a matrix?



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- There has also been some analysis based on Monte Carlo suggesting that 4 dimensional spacetime is energetically favorable over other dimensionality.
- This emergence of spacetime from a matrix model is very much in line with the philosophy that spacetime is a derived concept.
The world is a matrix?



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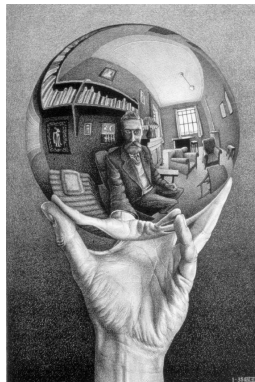
Area law of black hole entropy

$$S = \frac{A}{4}$$

leads to the Holographic Hypothesis: ('t Hooft, Susskind)

- Gravity in some region of space is completely described by a set of fundamental degree of freedom living on the boundary of the region.

According to holography, quantum gravity, that is spacetime itself and together with its quantum properties, are derived from a set of more fundamental degree of freedom living on the boundary.

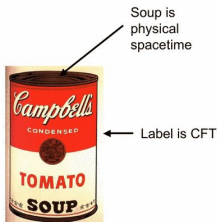


A fundamental theory of quantum gravity should incorporate this property.

Realization of Holography: AdS/CFT correspondence

In 1997, Maldacena argued from string theory that a concrete realization of holography:

- String theory on spacetimes $AdS \times S^5$
=
CFT living on the boundary R^4 at infinity.



- AdS = Anti de Sitter spacetime - the maximally symmetric spacetime with negative curvature
 CFT = Conformal Field Theory - ordinary nongravitational Yang-Mills gauge field theory which is conformally invariant.

- This duality was derived from a decoupling limit of the near horizon physics of D3-branes.
So far there is no proof, but lot of evidences supporting it.
- And there has been many interesting applications:
 - i. AdS/plasma: properties of high temperature nuclear matter: quark/gluon plasma,
 - ii. AdS/CDM: condensed matter physics
 - iii. AdS/nuclear: nuclear physics
 - ...

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It is natural to consider the higher form generalization.

- For example, for a single M5-brane, there is a $U(1)$ 2-form gauge field potential $B_{\mu\nu}$ living on the worldvolume. When N M5-branes are put together, one expect to have an enhancement of the gauge symmetry.

But it is a puzzle what it is!

- The construction of the gauge syjmmetry turns out to be very difficult. For example, it is extremely difficult to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B_{\mu\nu}^a = \partial_\mu \Lambda_\nu^a - \partial_\nu \Lambda_\mu^a + (?), \quad H_{\mu\nu\lambda}^a = \partial_\mu B_{\nu\lambda}^a + \partial_\nu B_{\lambda\mu}^a + \partial_\lambda B_{\mu\nu}^a + (?).$$

to have nontrivial self interaction.

- Turns out the non-abelian terms “(?)” have to be non-local. The following generalized gauge symmetry has been proposed:

$$\delta B_{\mu\nu}^a = D_\mu \Lambda_\nu^a - D_\nu \Lambda_\mu^a, \quad D_\mu = \partial_\mu + A_\mu.$$

The gauge field is auxillary and does not carry any propagating degree of freedom (as required by supersymmetry!):

$$F_{\mu\nu} = \int dx_5 \tilde{H}_{\mu\nu}, \quad \text{where } \tilde{H}_{\mu\nu} := \frac{1}{6} \epsilon_{\mu\nu\alpha\beta\gamma} H^{\alpha\beta\gamma}.$$

(Chu 2011; Chu, Ko 2012)

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Main Lessons

There has been lot of progress in the understanding of geometry and symmetry in string theory! We briefly talked about:

- Small-Large distance duality and generalized uncertainty principle
- Noncommutative geometry
- Matrices and Spacetime
- Holography
- A generalization of Yang-Mills symmetry

There has also been many useful mathematical spinoffs.

Despite 40 years of effort, we are still far from having a complete satisfactory formulation of string theory.

Shall we be disappointed?

- It tooks many years to get to the modern formulation of quantum mechanics from the old quantum theory.

1859: Kirchhoff's statement of the blackbody radiation problem

1900: Planck formula $E = h\nu$.

1924: de Broglie put forward theory of matter waves

1925: Schrodinger equation and Heisenberg matrix mechanics

1927: Heisenberg uncertainty principle

- Fermat last theorem (1637) was eventually solved in 1995.

Lessons

- It is likely that many results we have today may turn out to be too naive. But hopefully the lessons we learnt would guide us to the right principles for the formulation of the Theory of quantum gravity and Unification.
- It is quite amazing that string theory provides a framework in which some of the most fundamental questions (e.g. structure of spacetime) may be addressed.
String theory enables us to ask many questions that one cannot even dream of asking before. Some of these questions look crazy.

"We are all agreed that your theory is crazy. The question that divides us is whether it is crazy enough to have a chance of being correct" (Bohr).

But ... have we asked the right question? the most crazy question?

Expect the
Unexpected



It is your turn!