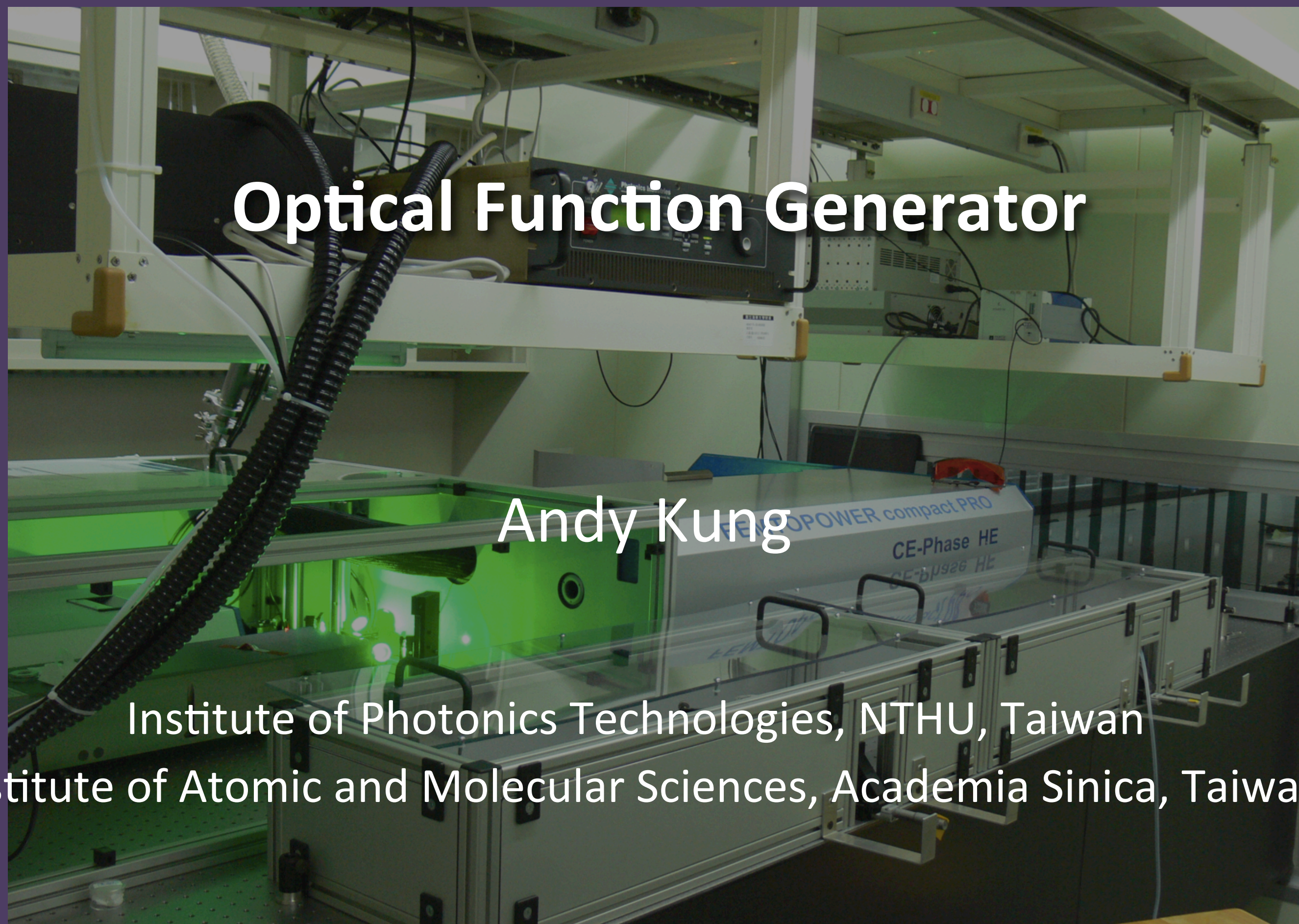
Optical Function Generator



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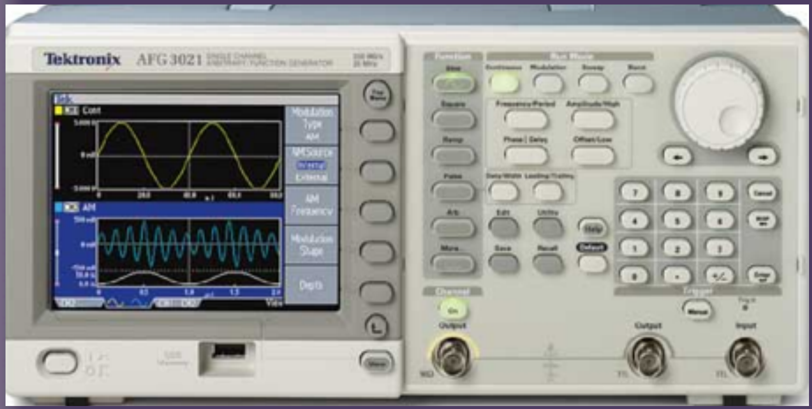
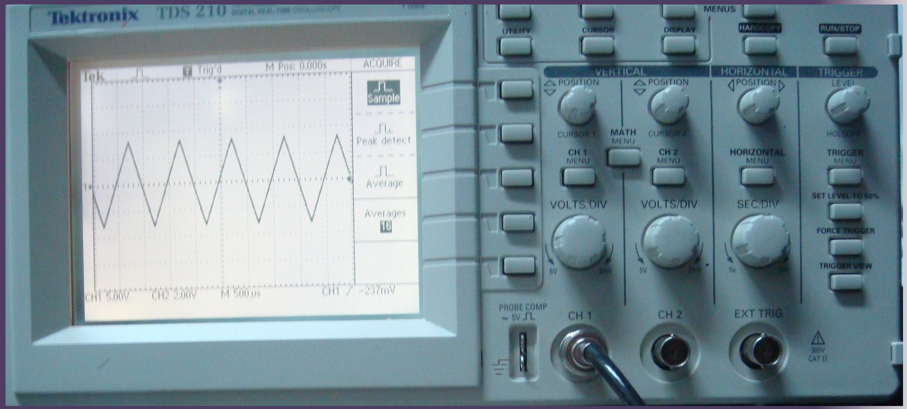
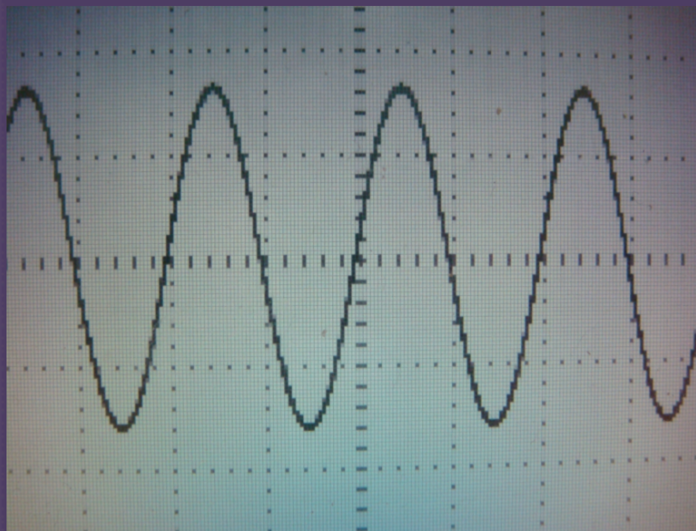
Institute of Atomic and Molecular Sciences, Academia Sinica, Taiwan

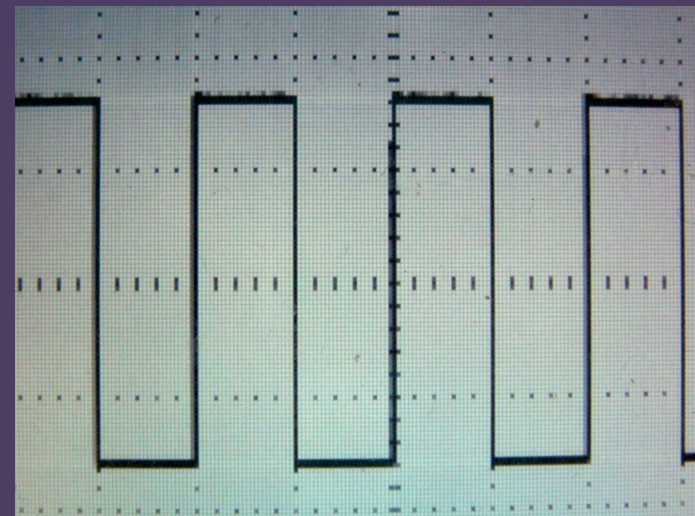
Physics Department, NTHU, Jan 2, 2013

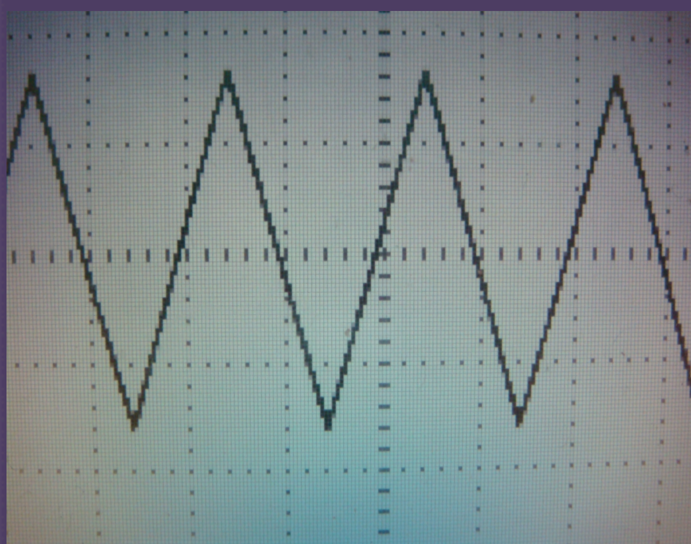
A function generator (函數產生器) is used in many physical science laboratories to generate a wide variety of synthesized electrical signals such as the sine, square, triangle and ramp functions. These signals are used for testing and diagnostic applications.

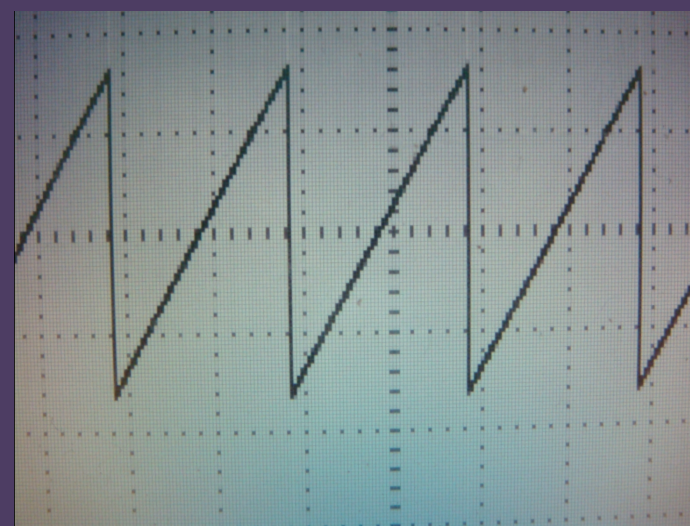




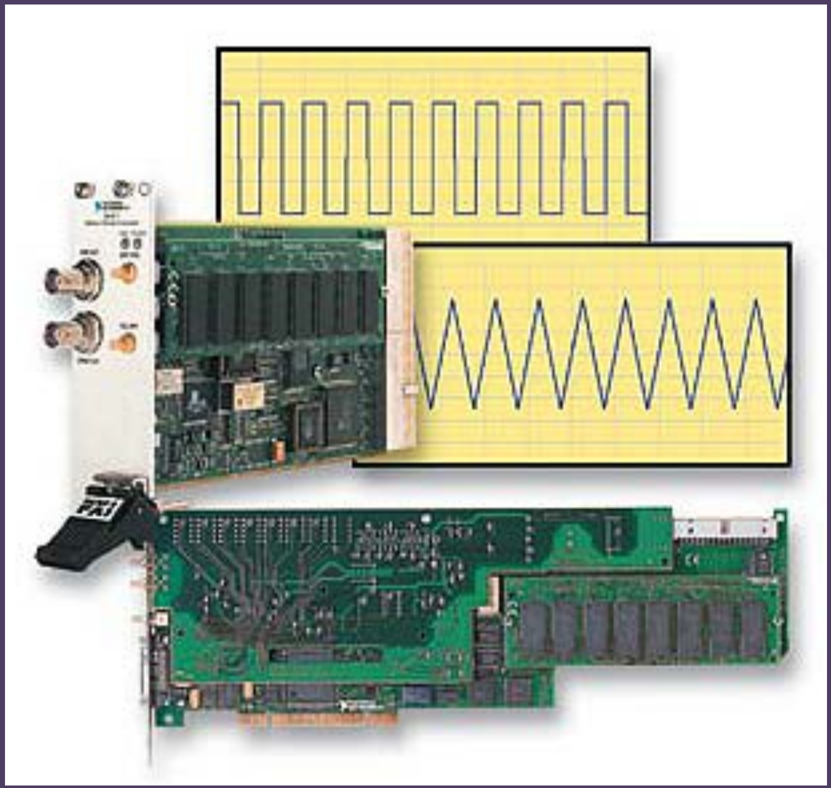

sine


square


triangle


sawtooth

RF and Microwave function generators

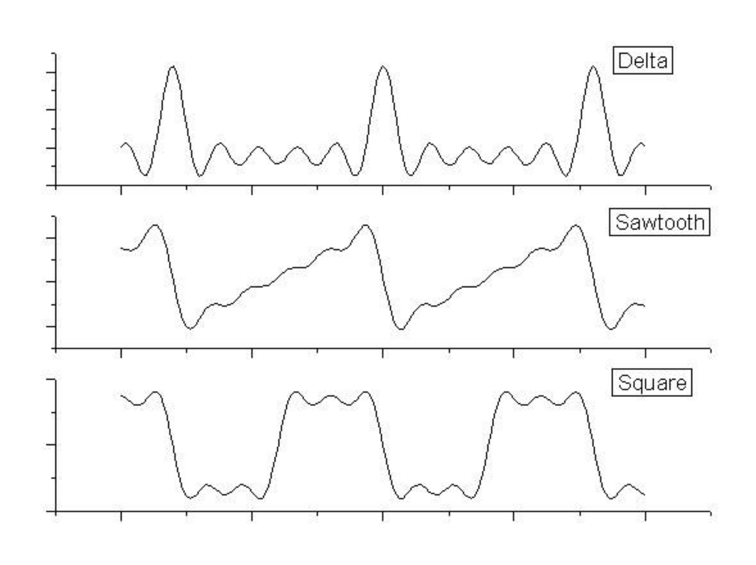


Their speed are limited by the speed of IC circuits to <100GHz.

Challenge :

Synthesize various forms of ultrashort pulses

in the optical frequency (10^{15} Hz) regime

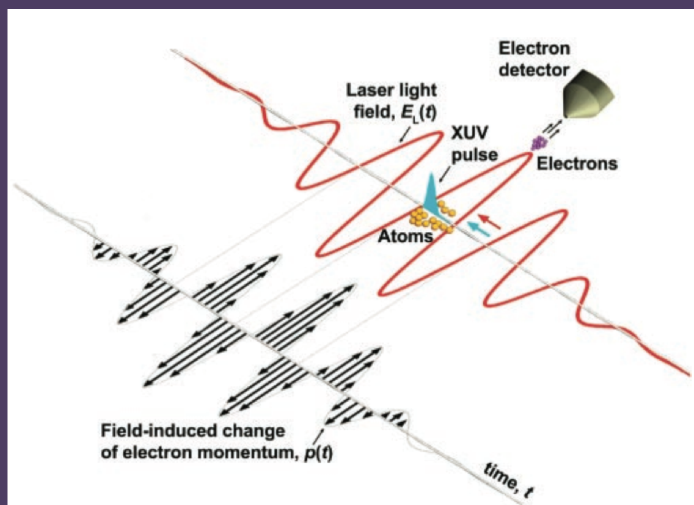


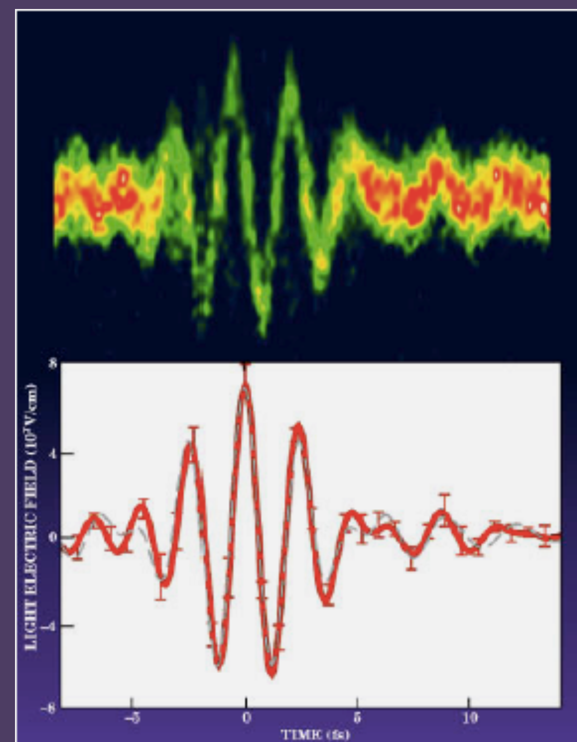
PHYSICS TODAY   October 2004

Search and Discovery

Attosecond Bursts Trace the Electric Field of Optical Laser Pulses

The familiar textbook sketch of light's oscillating electric field can now be drawn directly from measurements.





Pump: 750 nm(10 fs), probe 93 eV(15 nm)

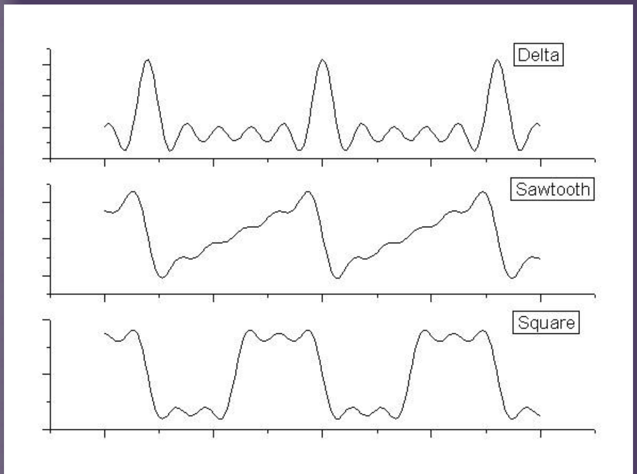
Science 305, 1267 (2004)

Wouldn't it be nice if we can have complete control of the motion of electrons in space and in matter, or just about anywhere?

Electrons are charged particles. Their motion follows changes in the electric field. To control electron motion, we want electric fields of an arbitrary shape that we can produce at will.

GOAL

Develop an Optical Function Generator



With these waveforms we hope that we can do using lightwaves what others have done with microwaves:

Control free/bound electron motion with light

LIGHTWAVE ELECTRONICS

Outline

Basic concept

Generation of broad spectral bandwidth

Phase control

Examples of optical waveform synthesis

Pitfalls and issues – possible solution

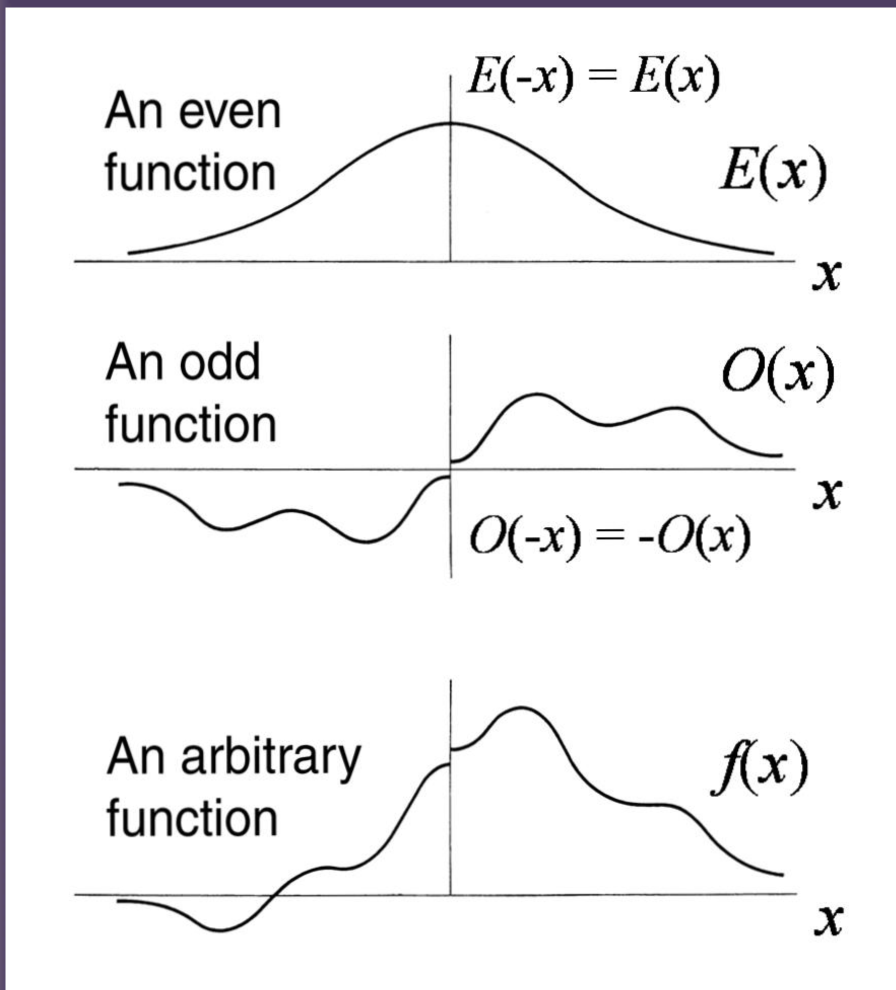
Basic Concept

Joseph Fourier

Fourier was obsessed with the physics of heat and developed the Fourier series and transform to model heat-flow problems.

Joseph Fourier 1768 - 1830

Any function can be written as the sum of an even and an odd function.



$$F(x) = E(x) + O(x)$$

Fourier series

So if $f(t)$ is a general function, neither even nor odd, it can be written:

even component odd component

$$f(t) = \frac{1}{\pi}\sum_{m=0}^{\infty} F_m \cos(mt) + \frac{1}{\pi}\sum_{m=0}^{\infty} F'_m \sin(mt)$$

where

$$F_m = \int f(t)\cos(mt)\,dt$$

and

$$F'_m = \int f(t)\sin(mt)\,dt$$

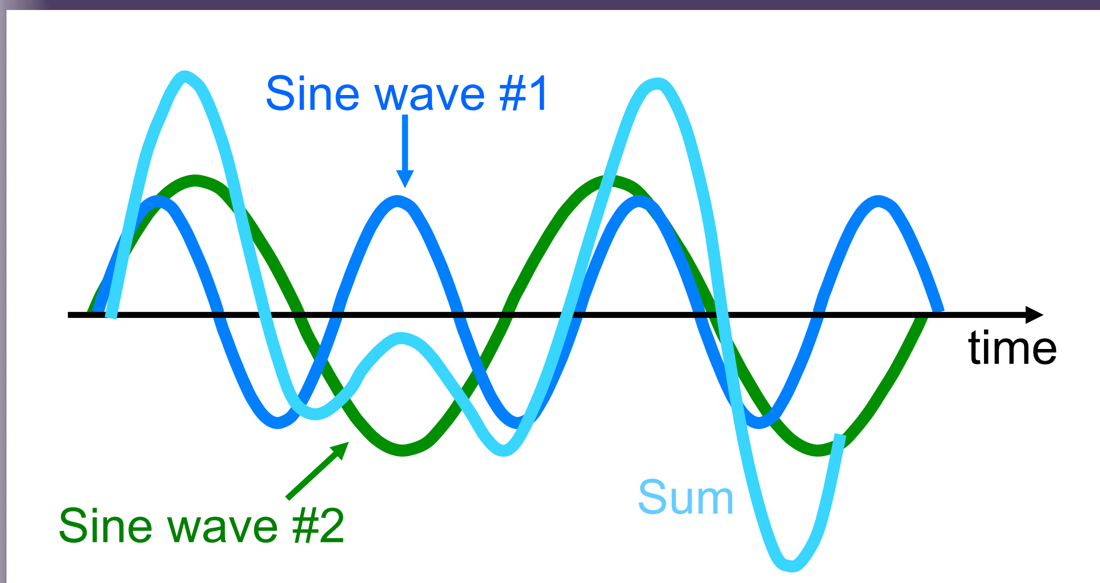
Waves are sums of sinusoids.

(a) Monochromatic light: sinusoidal wave

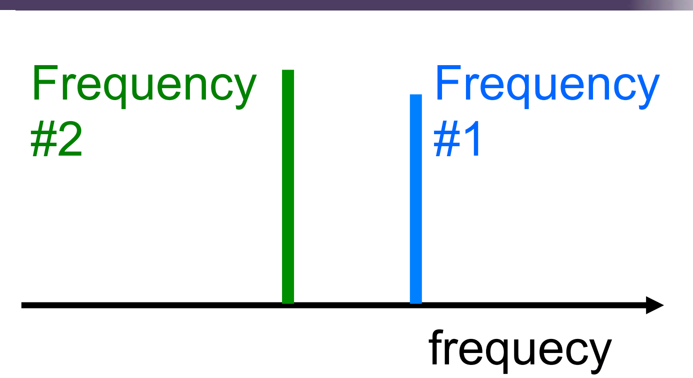
$$\boxed{E(t) = A(t)\cos(\omega t + \phi)}$$

(b) Beating of two waves, ω_1 and ω_2

$$\boxed{E(t) = A_1(t)\cos(\omega_1 t + \phi_1) + A_2(t)\cos(\omega_2 t + \phi_2)}$$



sum of two sine waves of different frequencies:



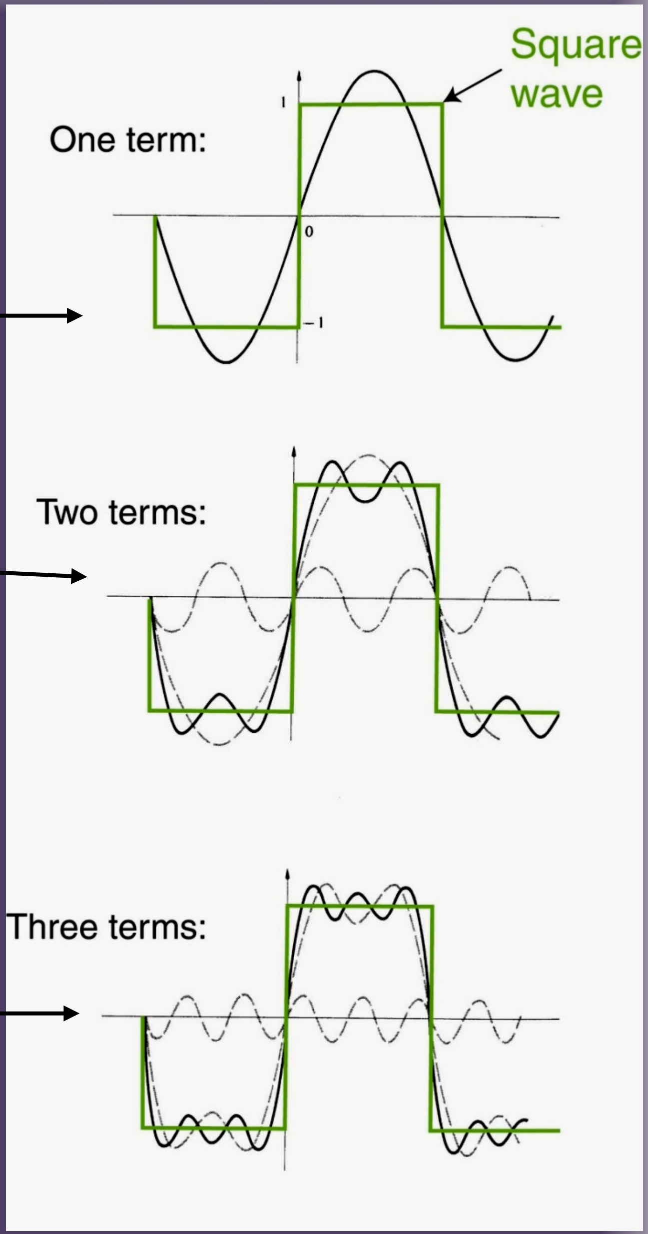
The spectrum of the two sine waves:

Fourier decomposing functions

$\sin(\omega t)$

$\sin(3\omega t)$

$\sin(5\omega t)$

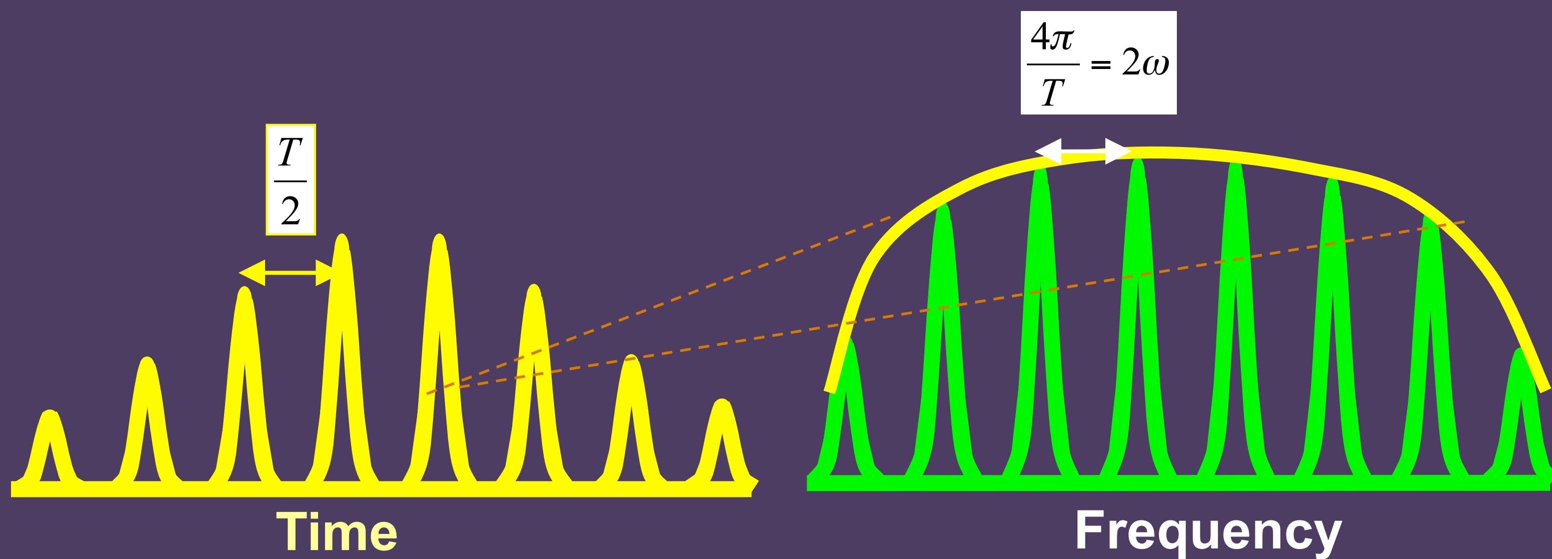


- Here, we write a
- **square wave** as
- a sum of sine waves.

Correlation between time and frequency

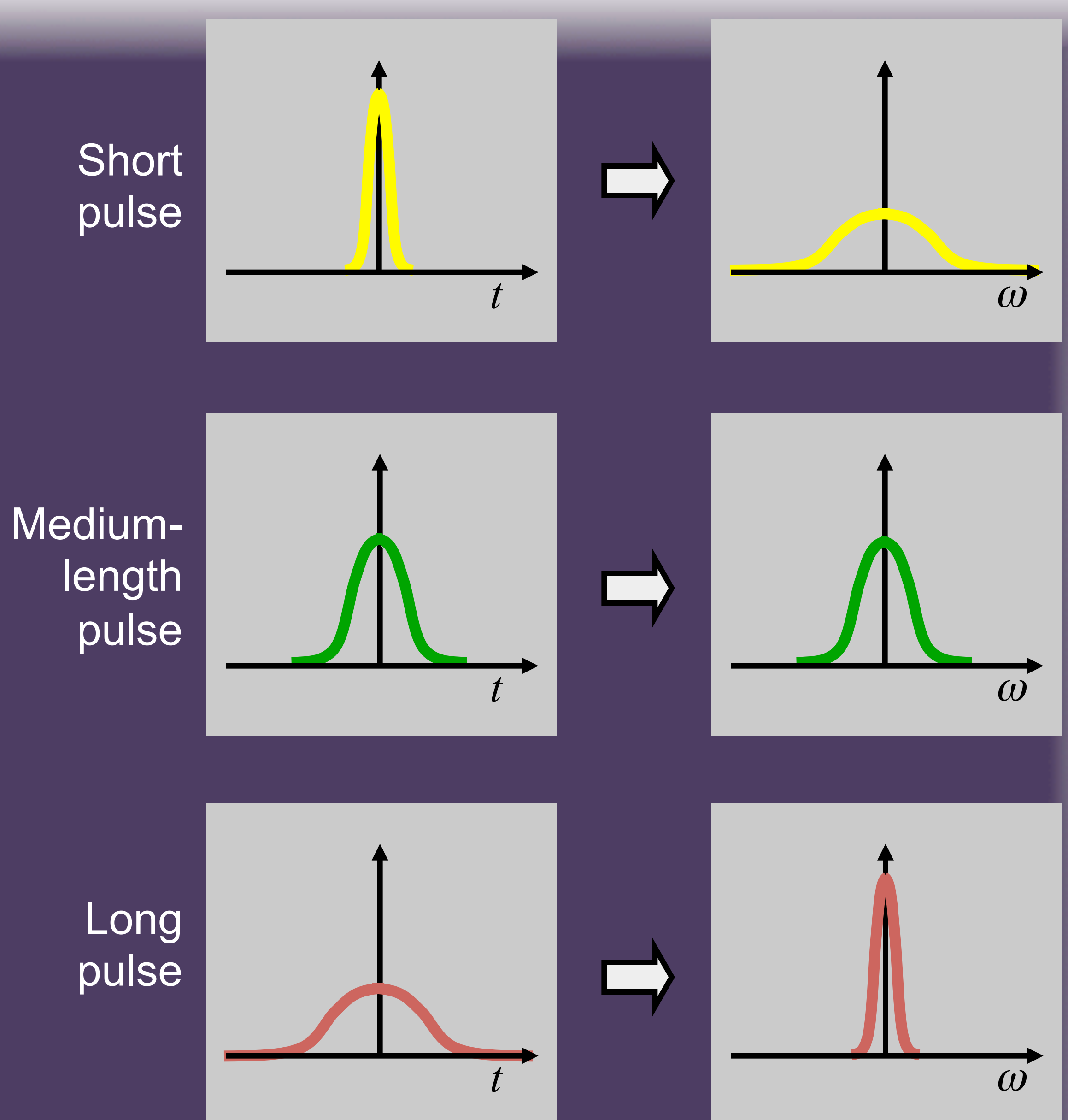
$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$$

Fourier transform: $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} d\omega$



2.6 fs ⟷ ***12,820 cm^{-1} , 780 nm***

The shorter the pulse, the broader the spectrum!

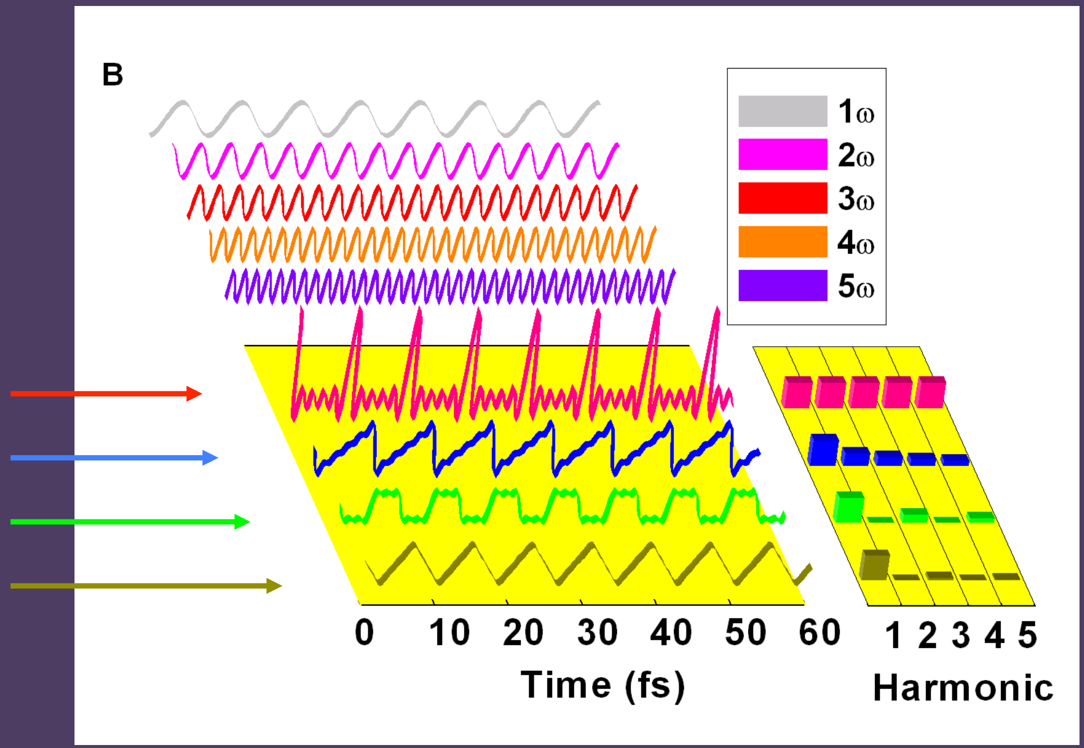


Our approach: Fourier Synthesis

Periodic waveforms are sums of sinusoidal waves

$$E(t) = \sum_n E_n(t) = \sum_n A_n(t)\cos(\omega_n t + \phi_n)$$

$\omega_n = n\omega_m$



Waveform synthesis

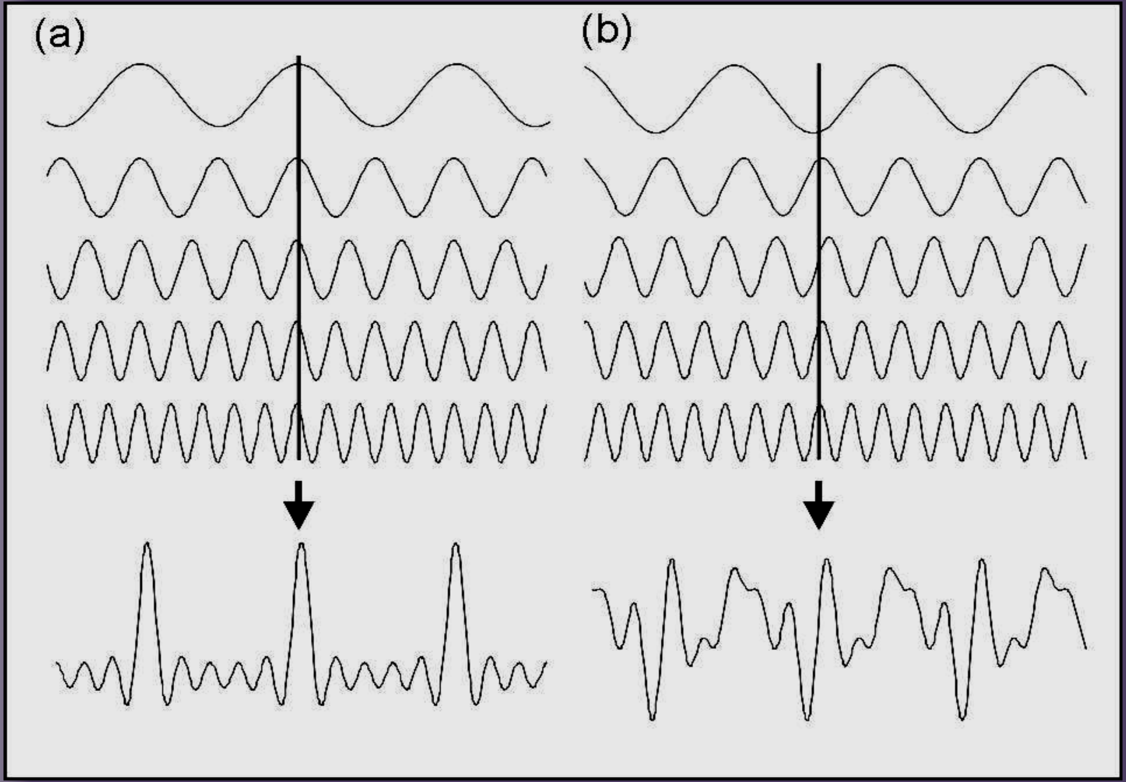
http://www.falstad.com/fourier/e-phase.html

Waveform	Formula	Normalized amplitude	CEP
Cosine	$\cos(n\omega t)$	1	NA
Sine	$\cos(n\omega t+\pi/2)$	1	NA
Single-cycle cosine	$\sum\cos(n\omega t)$, n=integer	{1, 1, 1, 1, 1, ...}	0
Single-cycle sine	$\sum\cos(n\omega t+\pi/2)$, n=integer	{1, 1, 1, 1, 1, ...}	$\pi/2$
Sawtooth	$(2/\pi)\sum\cos(n\omega t+\pi/2)/n$, n=integer	{1/1, 1/2, 1/3, 1/4, 1/5, ...}	$\pi/2$
Triangular	$(8/\pi^2)\sum\cos(n\omega t)/n^2$, n=odd	{1/1, 0, 1/9, 0, 1/25, ...}	0
Square	$(4/\pi)\sum(-1)^{(n-1)/2}\cos(n\omega t)/n$, n=odd	{1/1, 0, -1/3, 0, 1/5, ...}	0

Phase coherence

$$E(t) = \sum_n E_n(t) = \sum_n A_n(t)\cos(\omega_n t + \phi_n)$$

(a) In phase $\phi_n = n\phi_o$ (b) Random phases



In waveform synthesis, phase is everything.

To synthesize optical waveforms:

- Coherent pulses with broad bandwidth
- Properly align the phases in the spectrum
- Have a way to measure the pulses

A proposed sub-femtosecond pulse synthesizer using separate phase-locked laser oscillators

T.W. Hänsch

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Received 13 August 1990

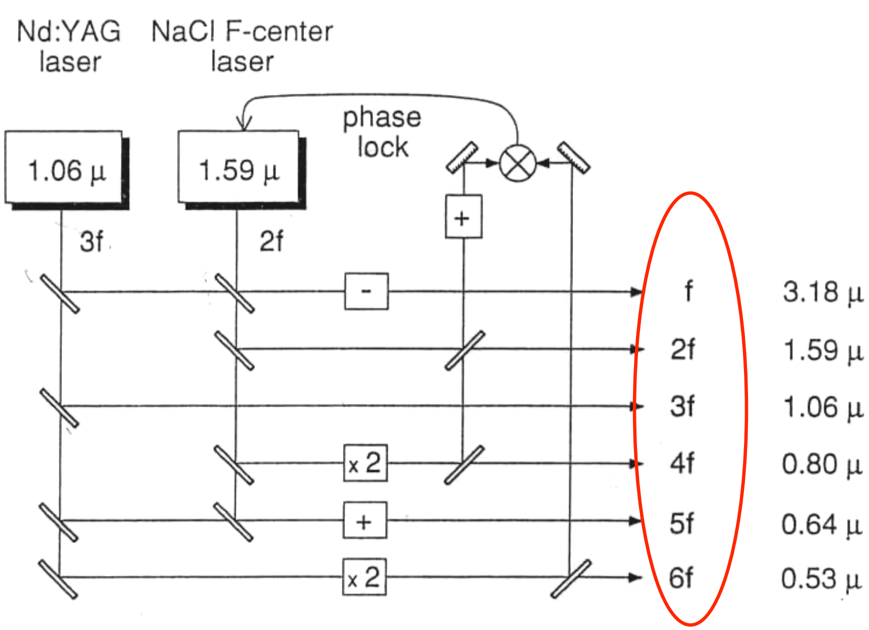
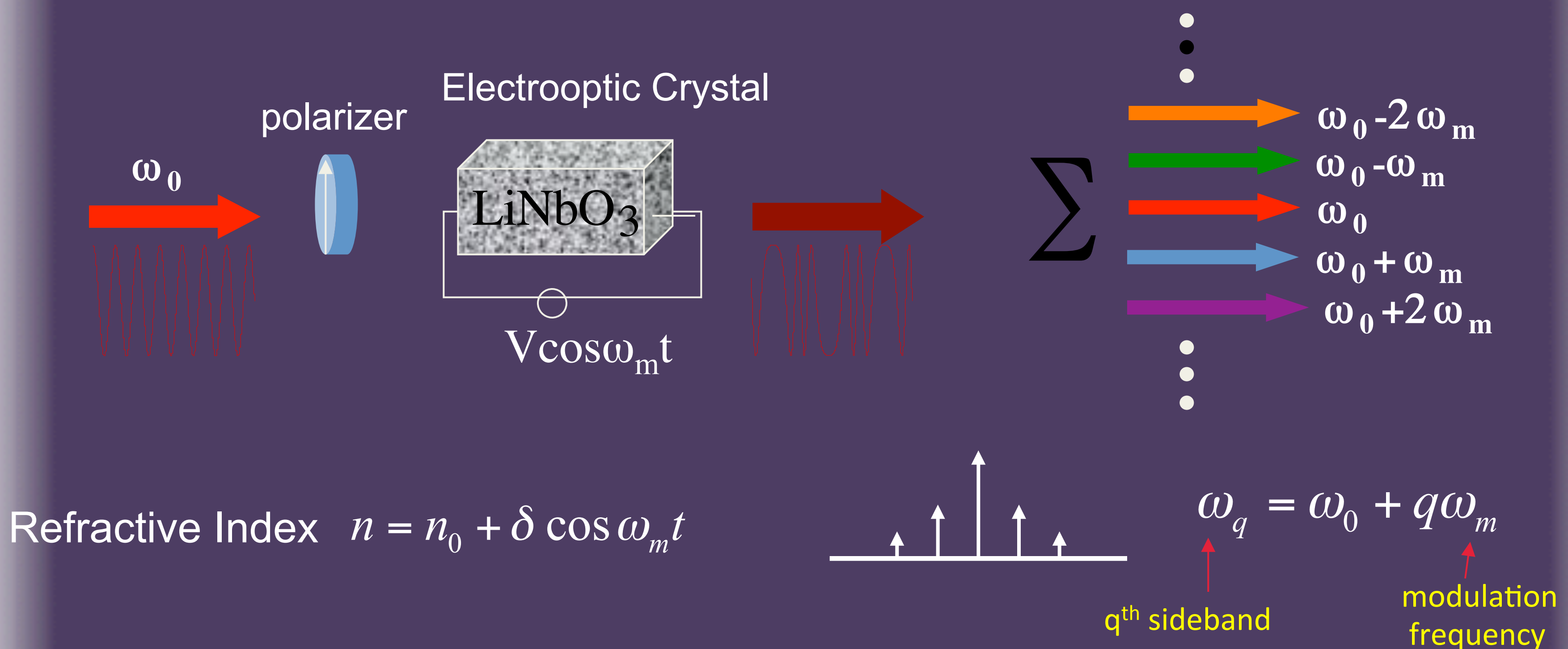
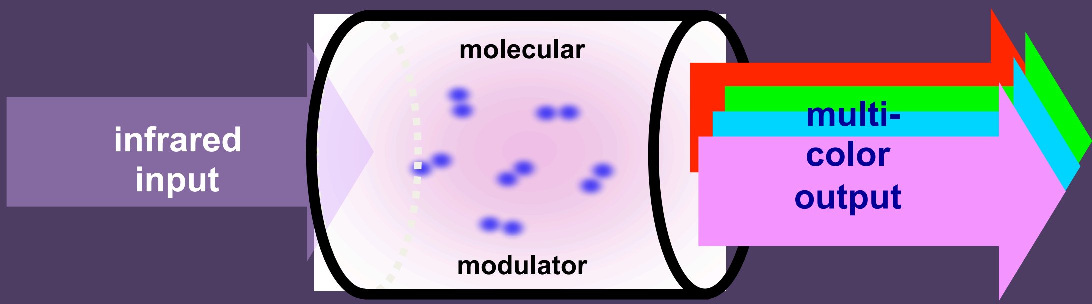


Fig. 3. Apparatus for generating a comb of six equidistant frequencies with two laser oscillators, employing sum- and difference-frequency mixing in nonlinear crystals.

Frequency comb expansion by modulation



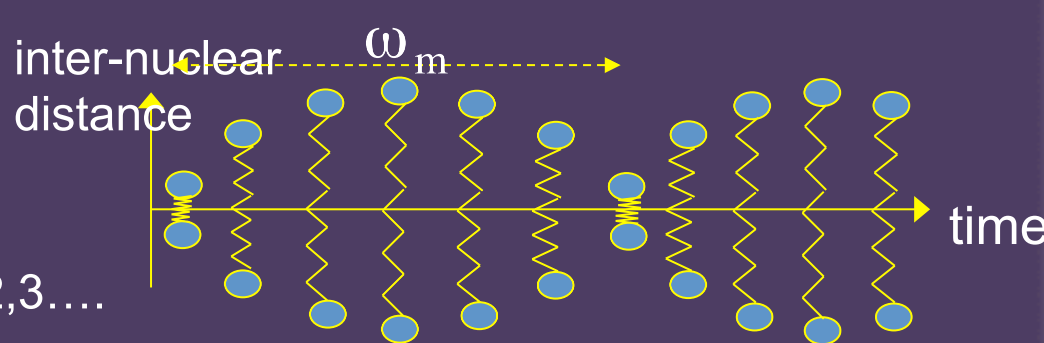
Bandwidth expansion by molecular modulation



Refractive Index

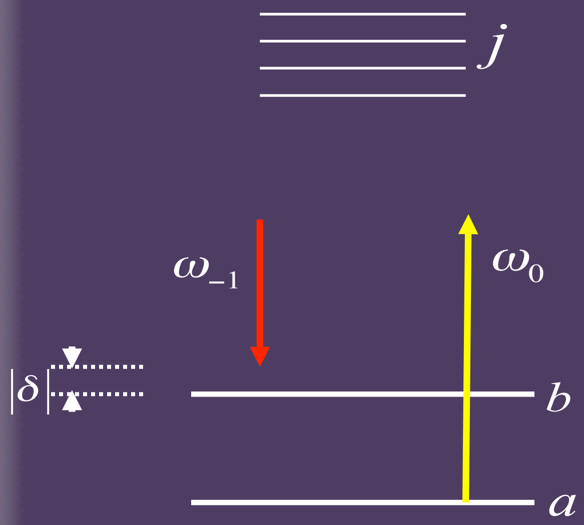
$$n = n_0 + \delta \cos \omega_m t$$

$\omega_q = \omega_0 + q\omega_m$ q= -2,-1,0,1,2,3….



S. E. Harris and A. V. Sokolov, Phys. Rev. A **55**, R4019 (1997);
S. E. Harris and A. V. Sokolov, Phys. Rev. Lett. **81**, 2894 (1998).

Octave-spanning frequency comb generated by the Raman technique (molecular modulation)



Harris and Sokolov, Phys. Rev. A (R)(1997)

$$\frac{\partial \rho_{aa}}{\partial \tau} = i\left(\Omega_{ab}\rho_{ba} - \Omega_{ba}\rho_{ab}\right) + \gamma_{\parallel}\rho_{bb}$$

$$\frac{\partial \rho_{bb}}{\partial \tau} = -i\left(\Omega_{ab}\rho_{ba} - \Omega_{ba}\rho_{ab}\right) - \gamma_{b}\rho_{bb}$$

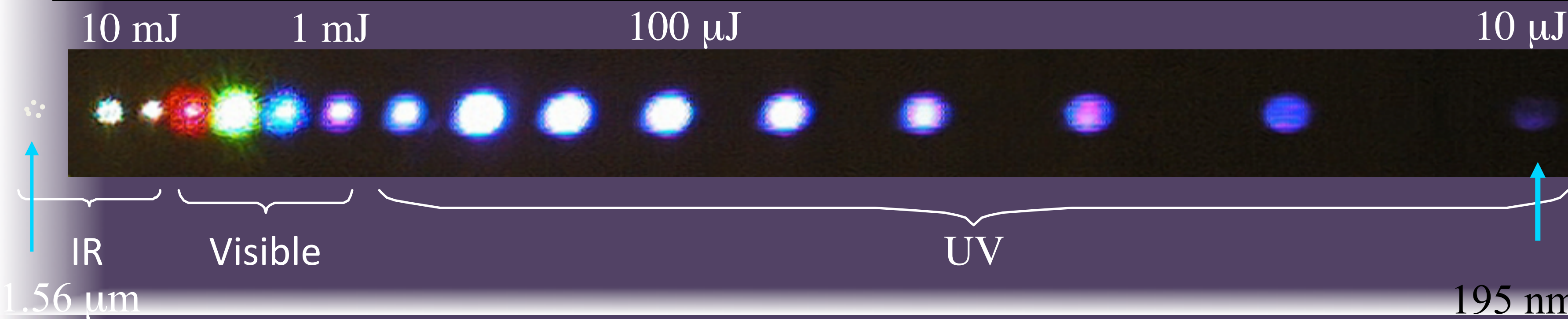
$$\frac{\partial \rho_{ab}}{\partial \tau} = i\left(\Omega_{aa} - \Omega_{bb} + \delta + i\gamma_{\perp}\right)\rho_{ab} + i\Omega_{ab}\left(\rho_{bb} - \rho_{aa}\right)$$

Detuning δ: adiabatic excitation, collinear sidebands

Absolute value of δ typically from 0.5 to 3 GHz

$$\frac{\partial E_q}{\partial z} = -j\eta\hbar\omega_q\left[N(a_q\rho_{aa}E_q + d_q\rho_{bb}E_q) + 0.666N(b_q^*\rho_{ab}E_{q-1} + b_{q+1}\rho_{ab}^*E_{q+1})\right]$$

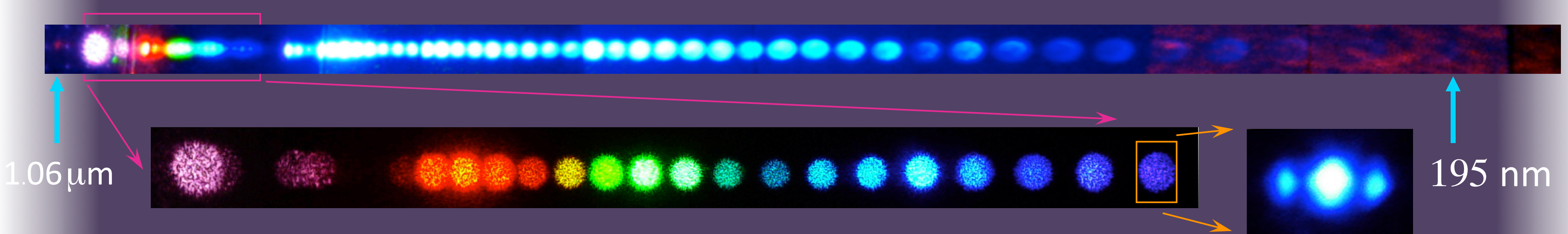
D_2 Vibration Spectra: 16 sidebands, spaced by 2994 cm^{-1}



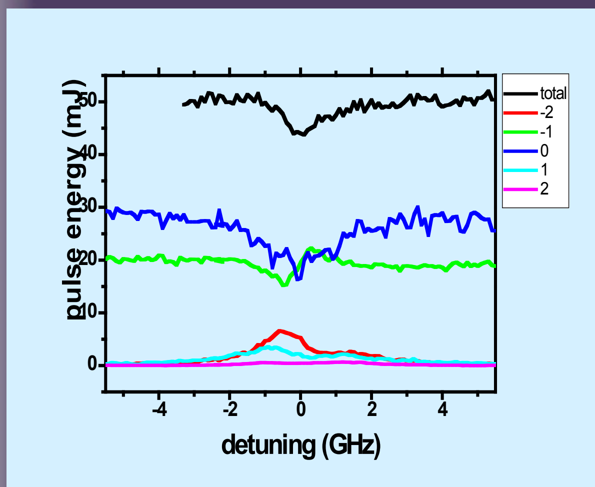
H_2 Rotation Spectra: 29 sidebands, spaced by 587 cm^{-1}

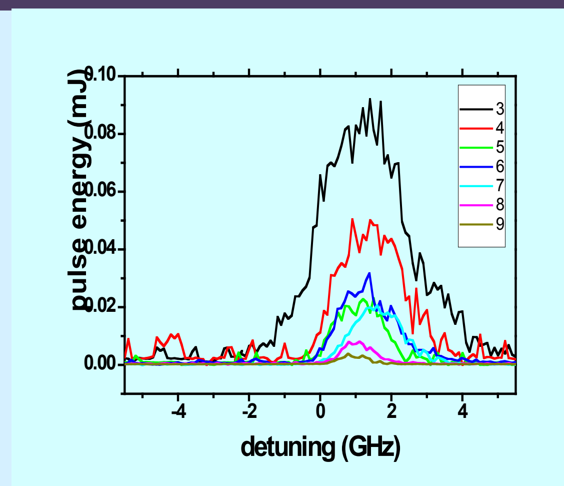
Phys. Rev. A (R)(1997)
Phys. Rev. Lett. 81 (1998)
Opt. Lett. 24 (1999)
Phys. Rev. Lett. 84 (2000)
Phys. Rev. Lett. 85 (2000)
Phys. Rev. A 63 (2001)
Phys. Rev. Lett. 91 (2003)
Phys. Rev. Lett. 93 (2005)

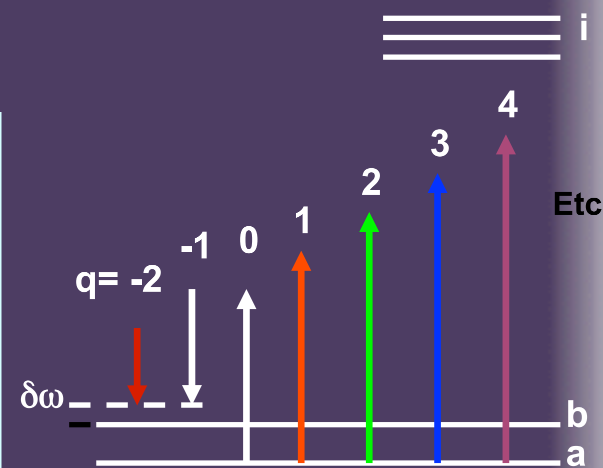
Multiplicative Spectra: ~ 200 sidebands, spaced by < 587 cm^{-1}



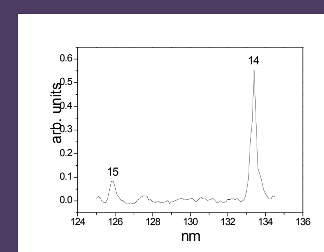
Raman comb generated in H_2







q	λ	
q = 3,	λ = 339.6	nm
6	238.6	nm
9	183.9	nm
12	149.6	nm
14	133.0	nm

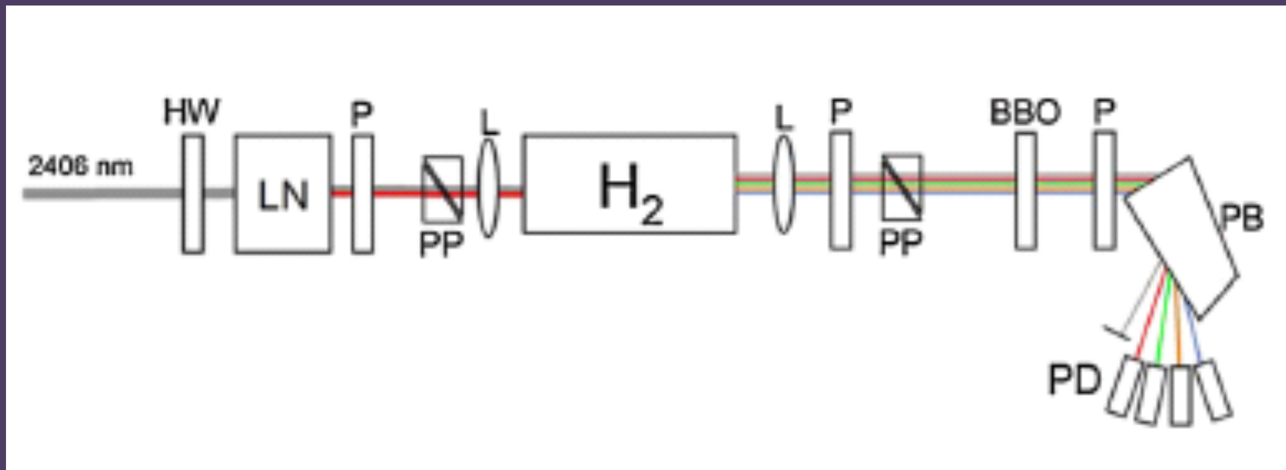


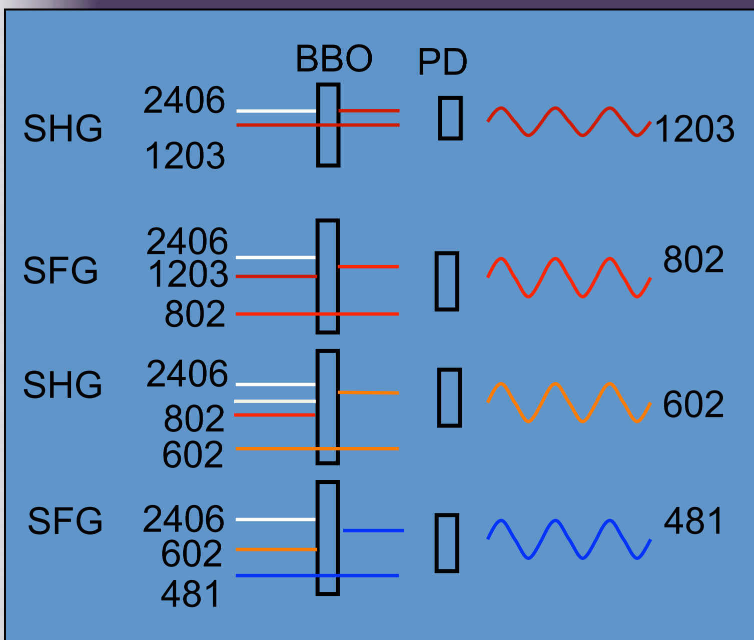
$\omega_q = \omega_0 + q\omega_m$ q= -2,-1,0,1,2,3….   CEP stable

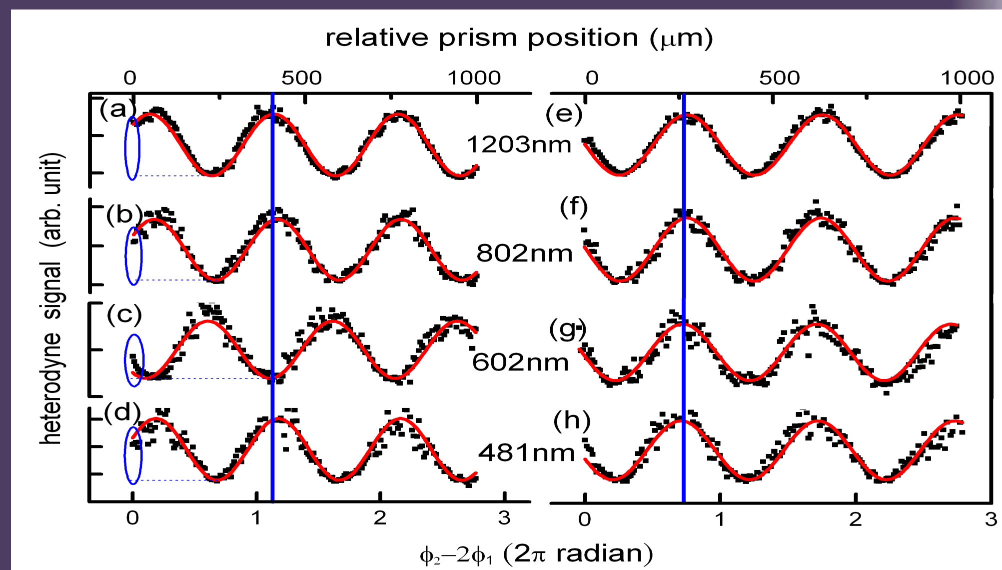
15th order at 126 nm observed

W.-J.Chen et al. PRL 100, 163906 (2008)

Phase detection : f-2f heterodyne



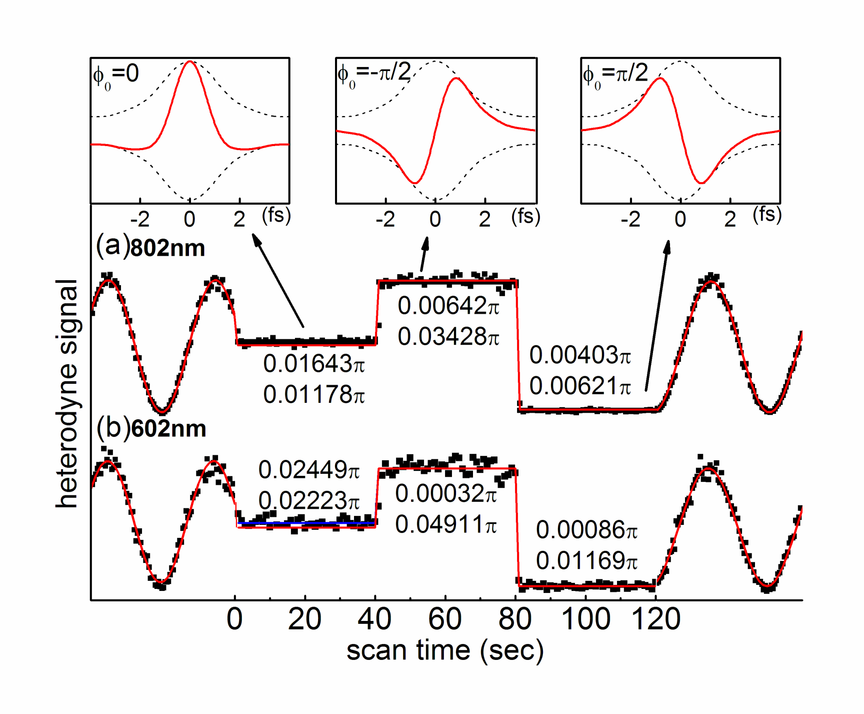




CEP controlled single-cycle pulses

Use ω, 2ω to drive molecular coherence

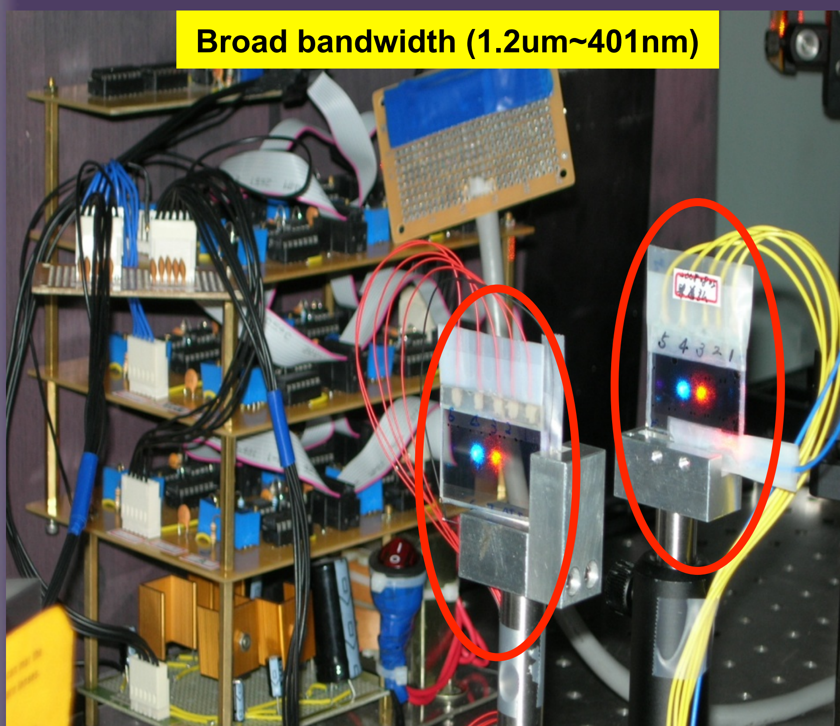
CEP setting accuracy <0.025π
CEP stability <0.05 π



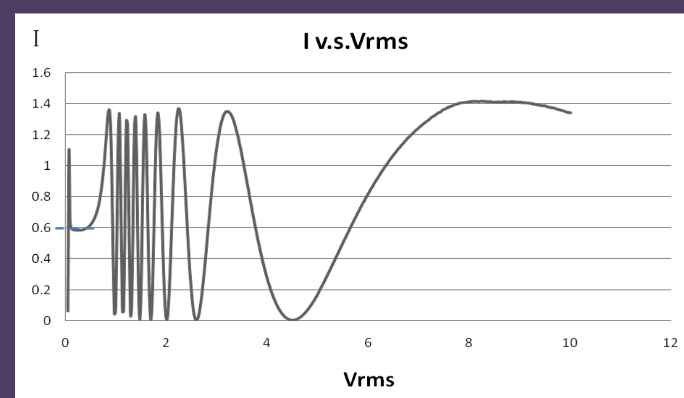
Hsieh et al., PRL 102, 213902 (2009)

The essential component: spatial liquid crystal modulators

For this idea to work we must have modulators that are broadband and can be configured to modulate either the phase or the amplitude of the input beam. We custom-make these modulators to match the size and spacing of our beam components. The LC material is E7 from Merck.

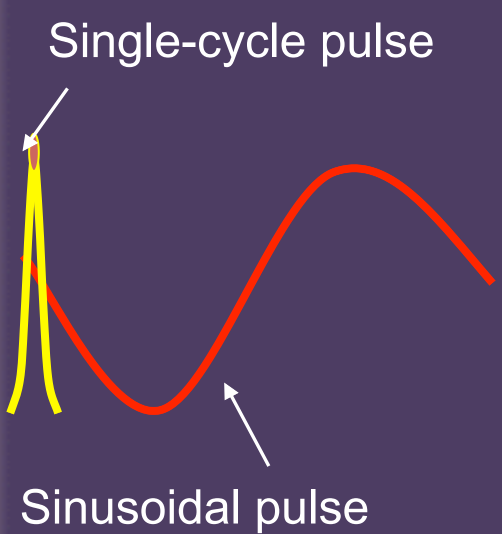


Now, our home-made liquid crystal modulator can cover wavelength from 2.4um to 401nm. Modulator for wavelength short than 344nm is now under testing.



Waveform Characterization

Field retrieval by *linear* cross-correlation of two fields $E_a(t)$ and $E_b(t)$



$$E_a(t) = \sum_n A_{an\omega} e^{i(n\omega t + \phi_{an\omega})}$$

$$E_b(t) = \sum_n A_{bn\omega} e^{i(n\omega t + \phi_{bn\omega} + \phi_{CEP})}$$

$$I(\tau) \propto \frac{1}{T} \int \sum_n \left[A_{an\omega}^2 + A_{bn\omega}^2 + 2 A_{an\omega} A_{bn\omega} \cos\left(\phi_{bn\omega} - \phi_{an\omega} + \phi_{CEP} + n\omega\tau\right)\right] dt$$

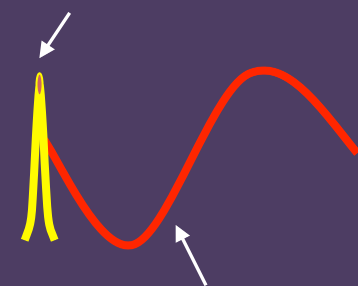
$$\propto \sum_n \left[A_{an\omega}^2 + A_{bn\omega}^2 + \boxed{2 A_{an\omega} A_{bn\omega} \cos\left(\phi_{bn\omega} - \phi_{an\omega} + \phi_{CEP} + n\omega\tau\right)}\right]$$

If $E_a(t)$ is the identity function, then the AC part of $I(\tau)$ is re{$E_b(\tau)$}

Shaper-assisted correlation of ultrafast waveforms

A. Galler and T. Feurer, Appl. Phys. B 90, 427-430 (2008).

Splitting a pulse optoelectronically, not optomechanically: the idea is to use a pulse shaper to split the pulse into two portions and electronically scan the delay time of one portion relative to the other without the need for a beam splitter and a delay line.



$$f(t) = \int F(\omega) e^{i\omega t} d\omega$$

$$f_a(t) = \frac{1}{2} f(t) + \frac{1}{2} f(t+\tau)$$

$$= \frac{1}{2} \int F(\omega) \left[e^{i\omega t} + e^{i\omega(t+\tau)} \right] d\omega$$

$$= \int \frac{1}{2} \left(1 + e^{i\omega\tau}\right) F(\omega) e^{i\omega t}\, d\omega = \int \cos\left(\frac{\omega\tau}{2}\right) e^{i\frac{\omega\tau}{2}} F(\omega) e^{i\omega t}\, d\omega$$

$$= \int F_a(\omega) e^{i\omega t}\, d\omega \quad \text{where} \quad F_a(\omega) \equiv \underbrace{\cos\left(\frac{\omega\tau}{2}\right)}_{\text{Amplitude}} \underbrace{e^{i\frac{\omega\tau}{2}}}_{\text{Phase}} F(\omega)$$

= amplitude modulation with a SLM

Linear cross correlation with a twist

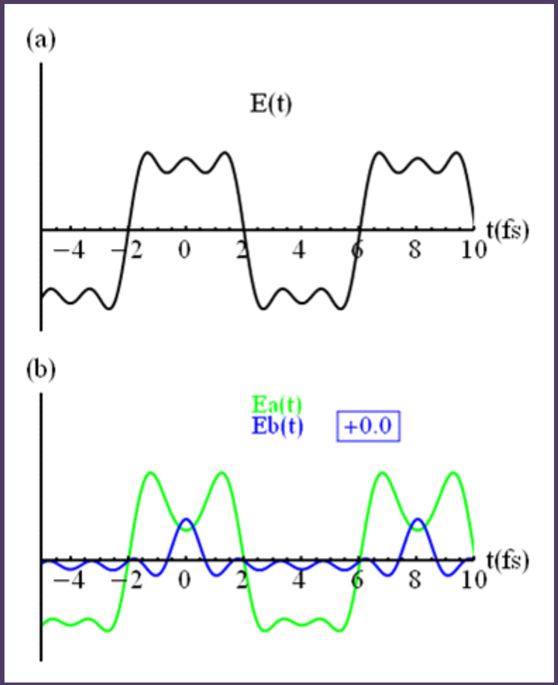
Shaper-Assisted Linear Cross-Correlation for optical field retrieval

SALC^2

We use pulse shaping to split our E field into two pulses to do the cross correlation.

Splitting a pulse optoelectronically, not optomechanically: the idea is to use a pulse shaper to split the pulse into two portions and electronically scan the delay time of one portion relative to the other without the need for a beam splitter and a delay line.

Note that pulse shaping require full knowledge of the phases. So monitoring of the phase of the pulses is essential for this technique to be meaningful.



$$E(t) = E_a(t) + E_b(t) = \{1, 0, -\frac{1}{3}, 0, \frac{1}{5}\}$$

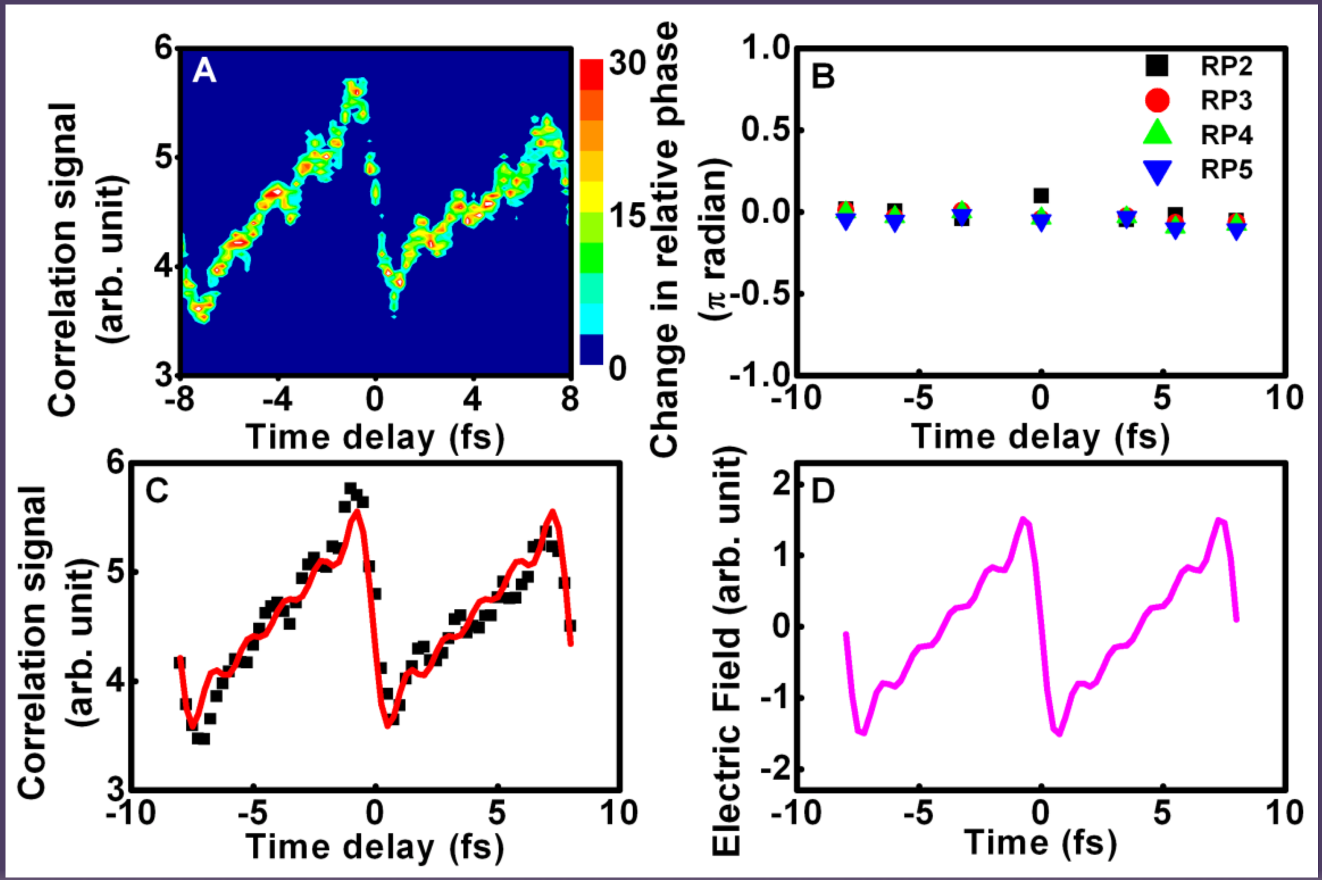
$$E_a(t) = \{\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}\}$$

$$E_b(t) = \{\frac{9}{10}, -\frac{1}{10}, \frac{13}{30}, -\frac{1}{10}, \frac{1}{10}\}$$

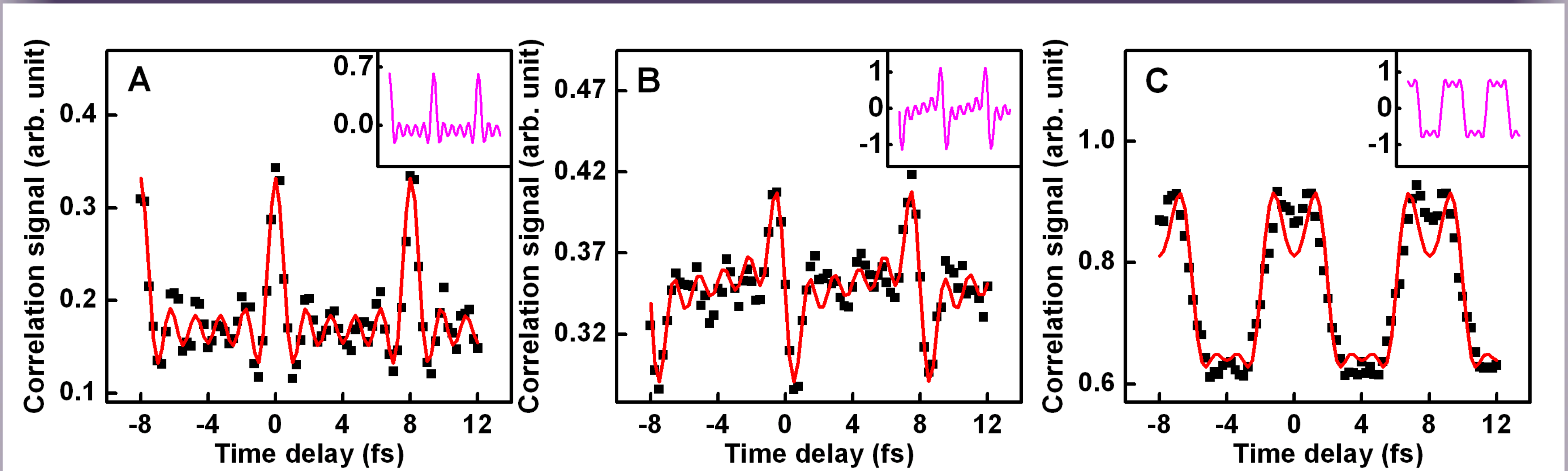
A. Galler and T. Feurer, Appl. Phys. B 90, 427-430 (2008).

Sawtooth waveform synthesized with 5 harmonic components

relative amplitudes {1,1/2,1/3,1/4,1/5} and CEP = $-\pi/2$



Instantaneous field waveforms synthesized with 5 harmonic components



Single-cycle cosine

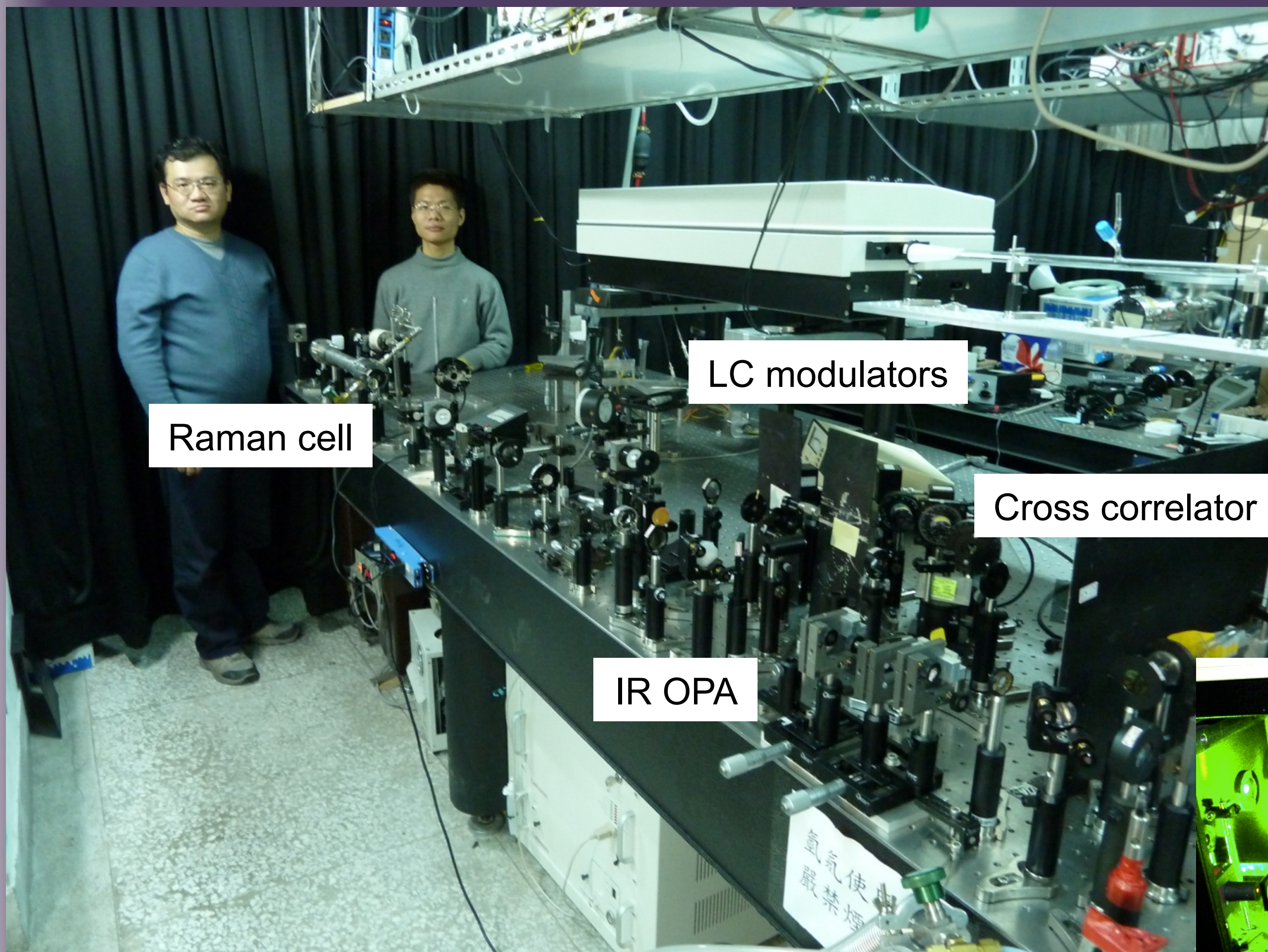
Single-cycle sine

Square wave

Peak to peak spacing: 8.02 fs
Narrowest field FWHM: 843 as
Envelope width: 1.5 fs

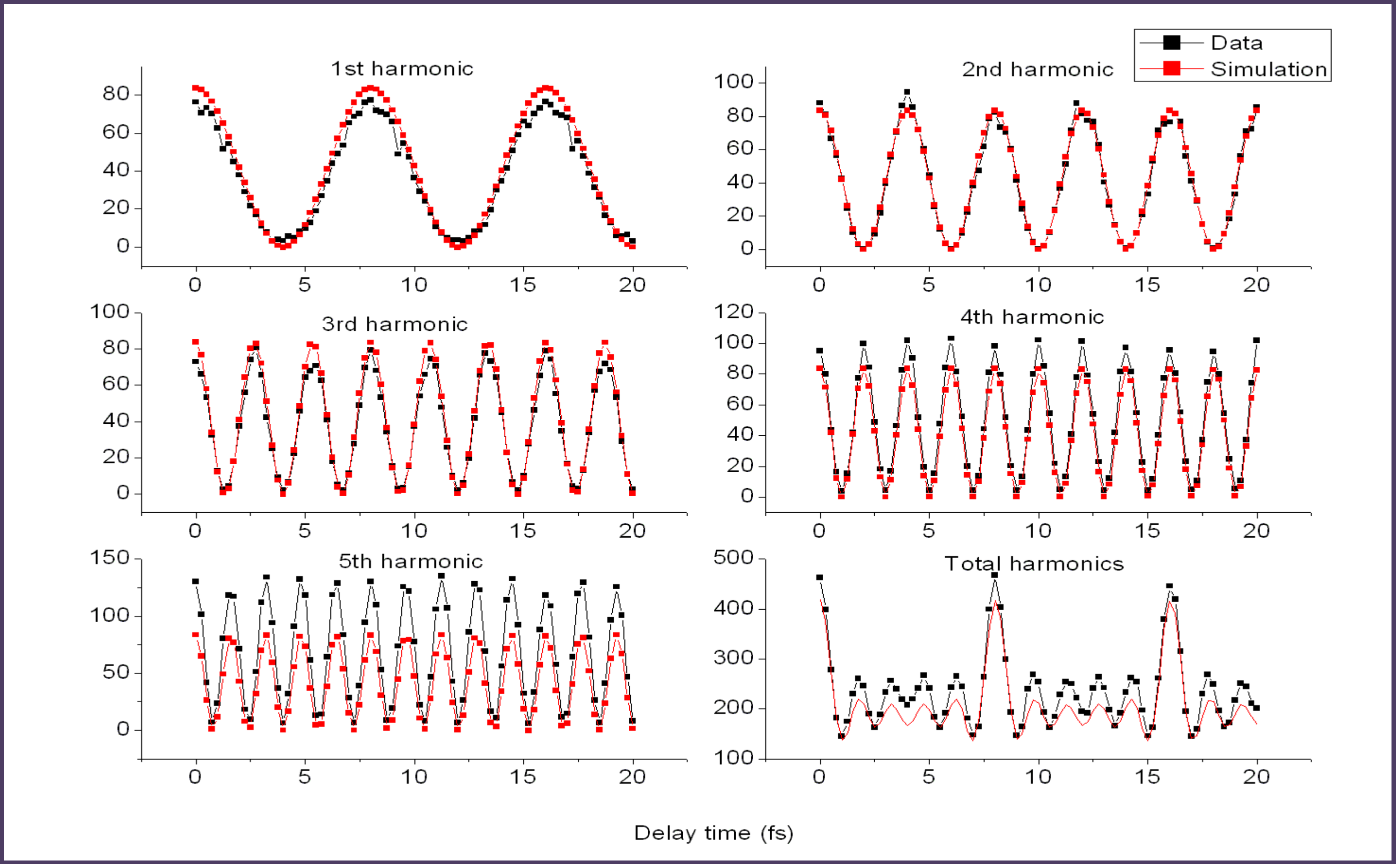
Chan et al, Science 331, 1165 (2011)

Optical waveform synthesizer set up



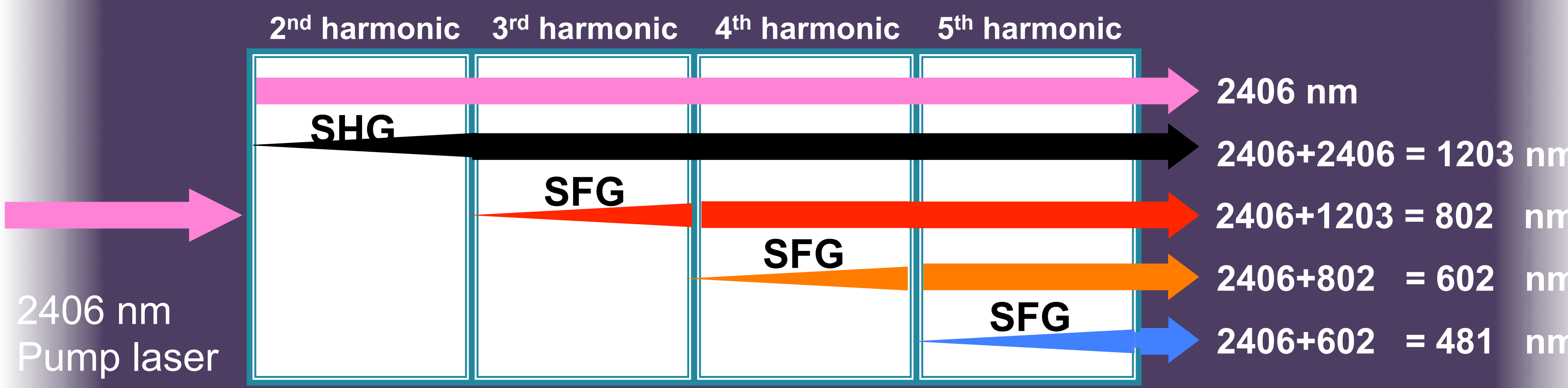
Laser source

Multimode or incoherent light gives the same result with SALC^2

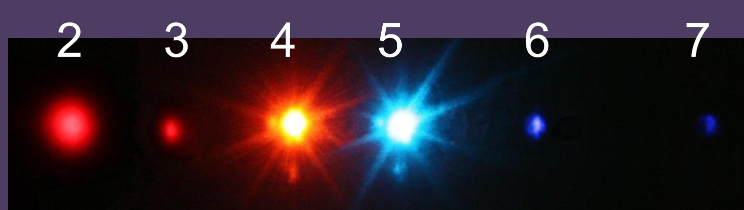


Purely adjusting the amplitudes
Phase information is not needed or retrieved

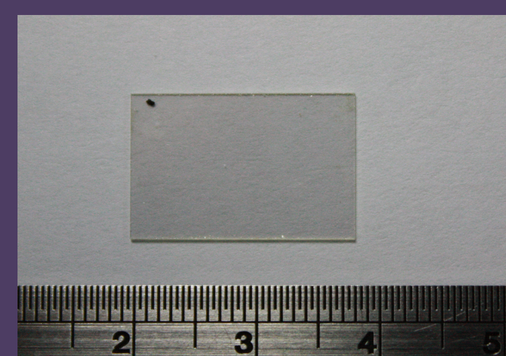
Cascaded harmonics generation in a monolithic crystal



1.8 cm long x 0.1 cm thick PPLT crystal

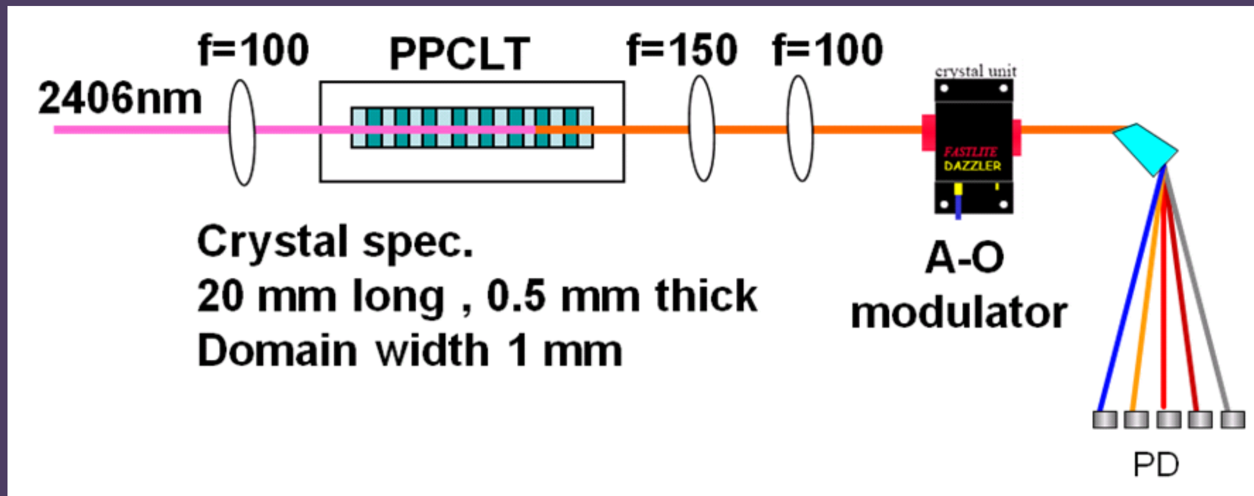


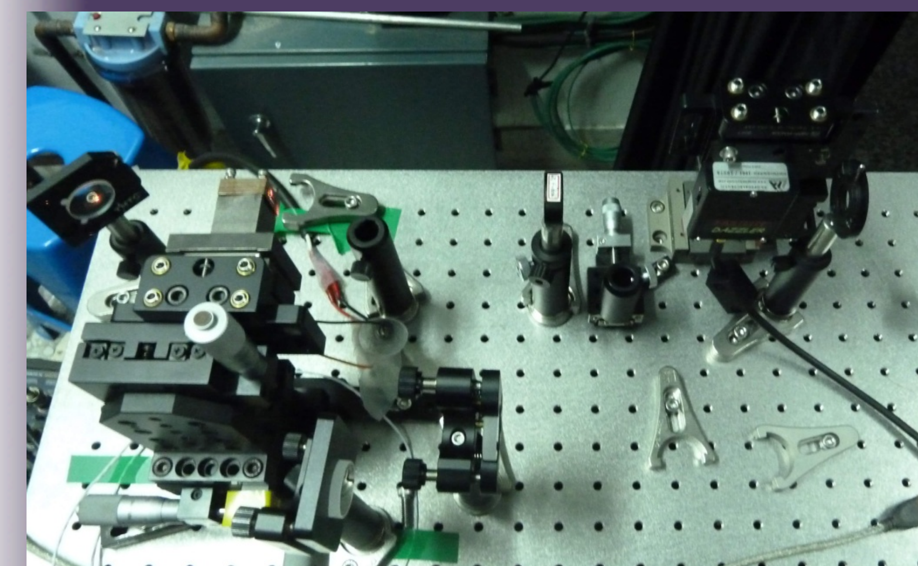
Up to the 7th harmonic has been observed

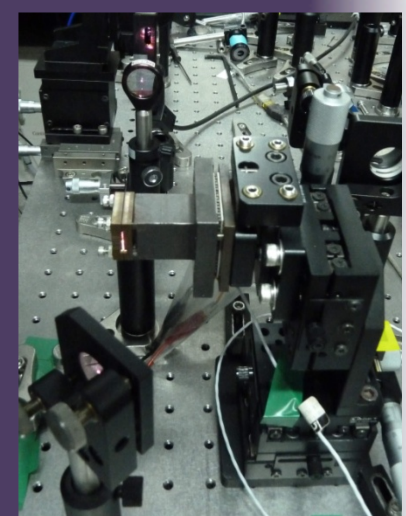


Optics Letters Nov 2009

Compact waveform synthesizer







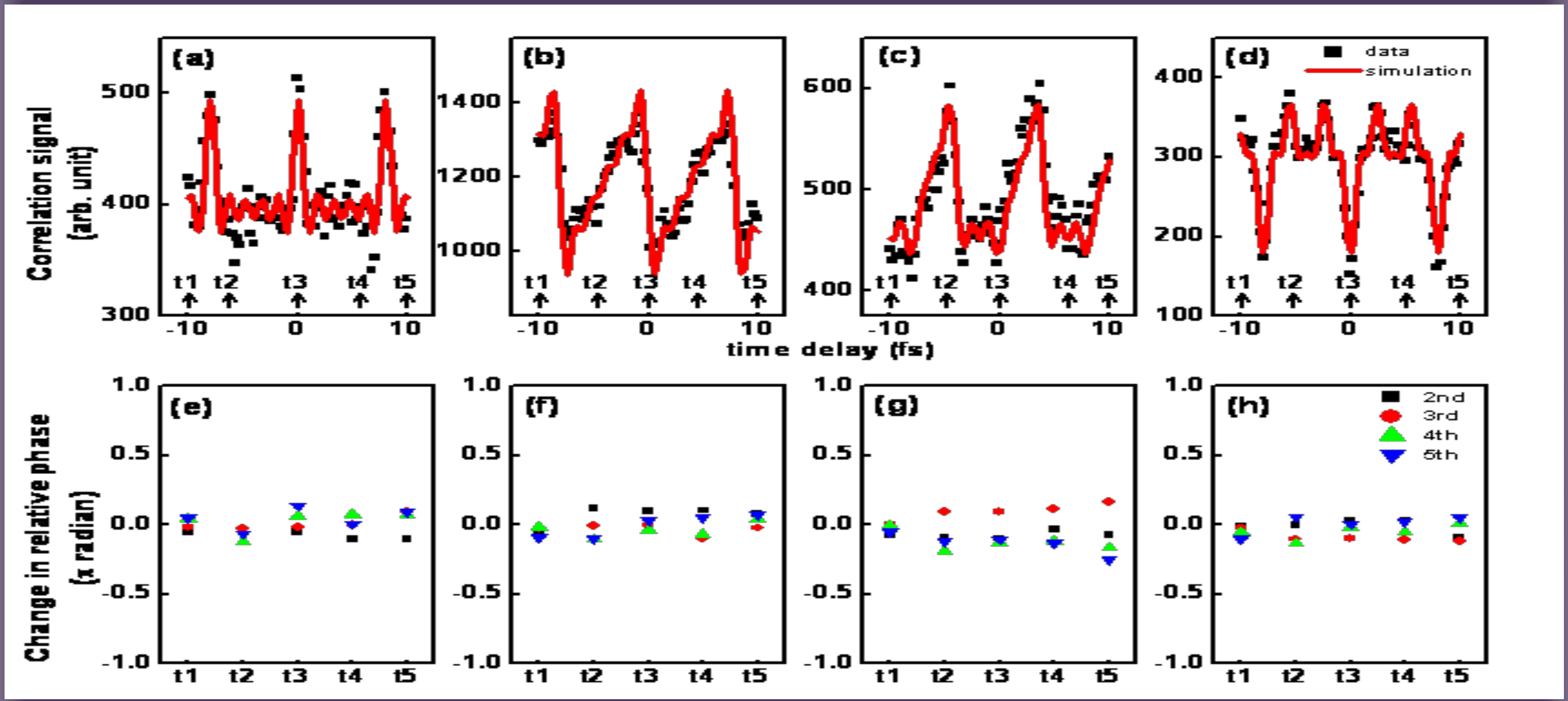
Breadboard set up

UB-AOM

PPLT

Optics Letters (2012)

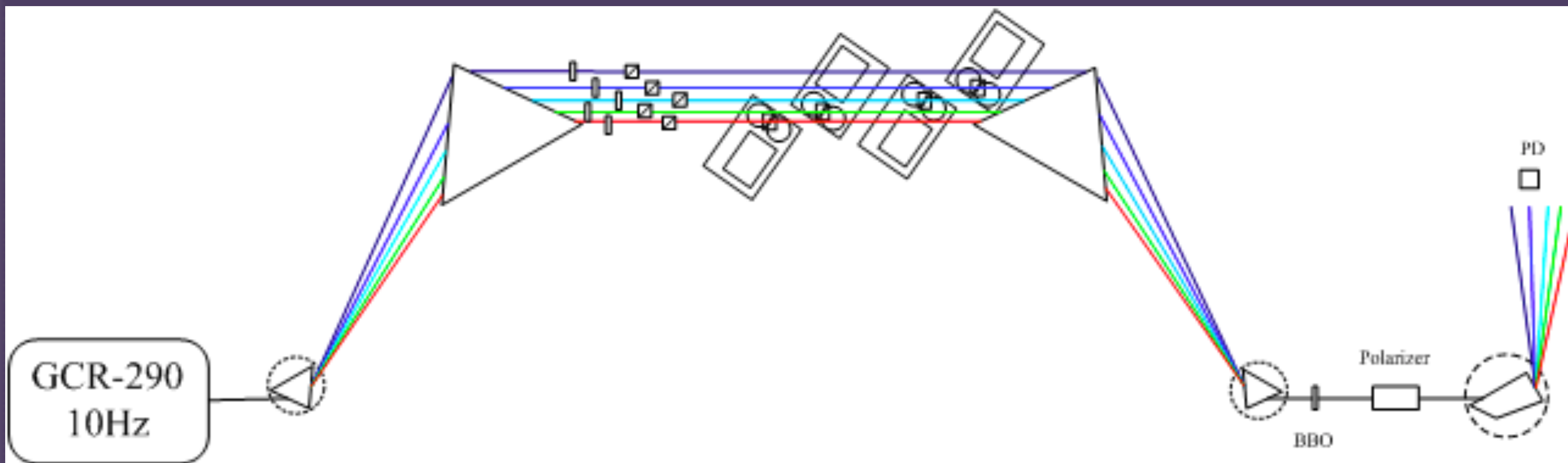
Instantaneous field waveforms synthesized

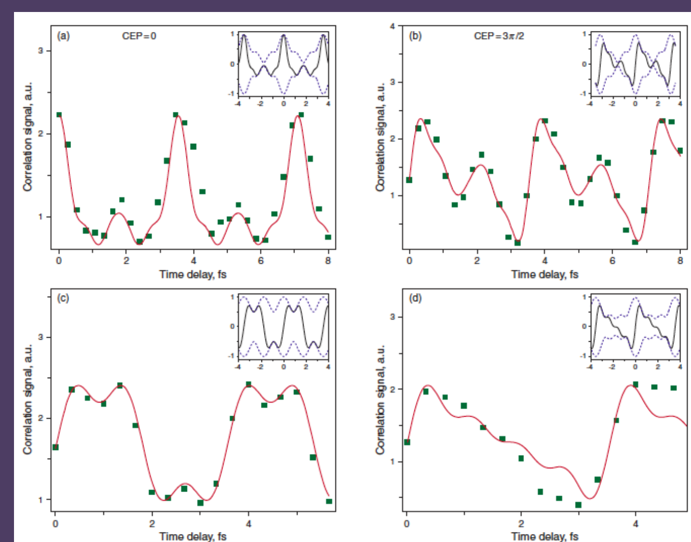


Single-cycle cosine | Sawtooth | Rectified sawtooth | Triple-peak

Peak to peak spacing: 8.02 fs
Narrowest field FWHM: 843 as
Envelope width: 1.5 fs

Synthesis with high power laser harmonics





Nd:YAG harmonics

Chen et. al. Laser Phys. Lett. 1-7 (2012)

Potential Applications

Attosecond flash photography

Quantum and coherent control

Lightwave electronics and communication(>100 THz)

HHG optimization using pulse shaping techniques

Measure dynamic Casimir force

Two main issues

What does the pulse train really look like?

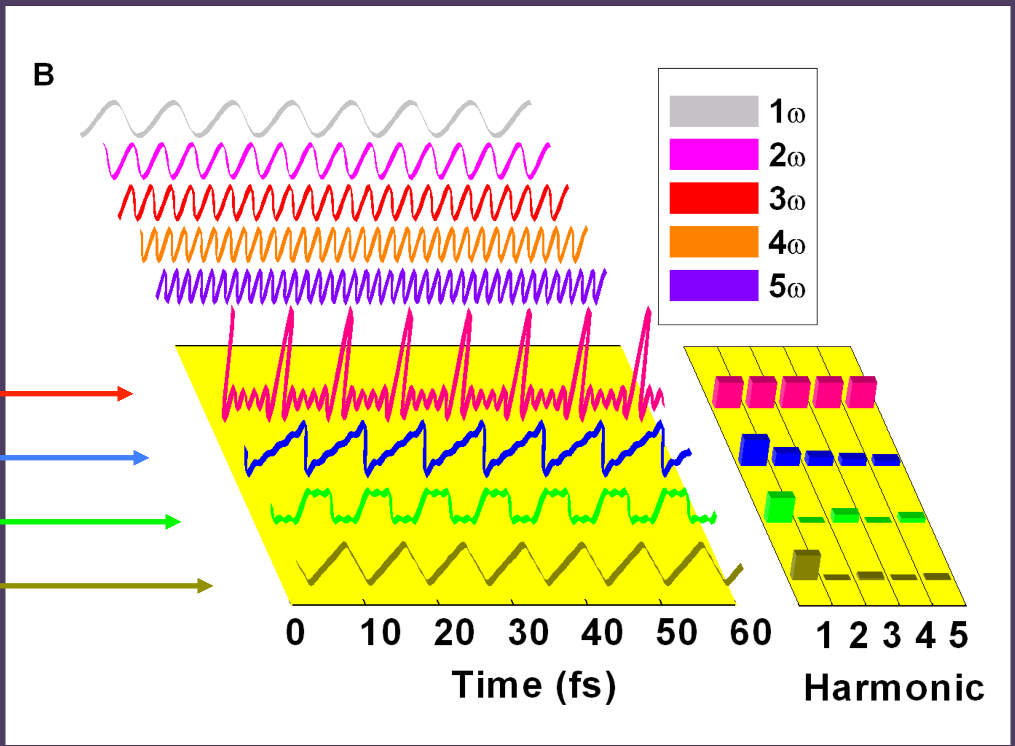
Are there potential faults in the characterization technique ?

Fourier synthesized pulses

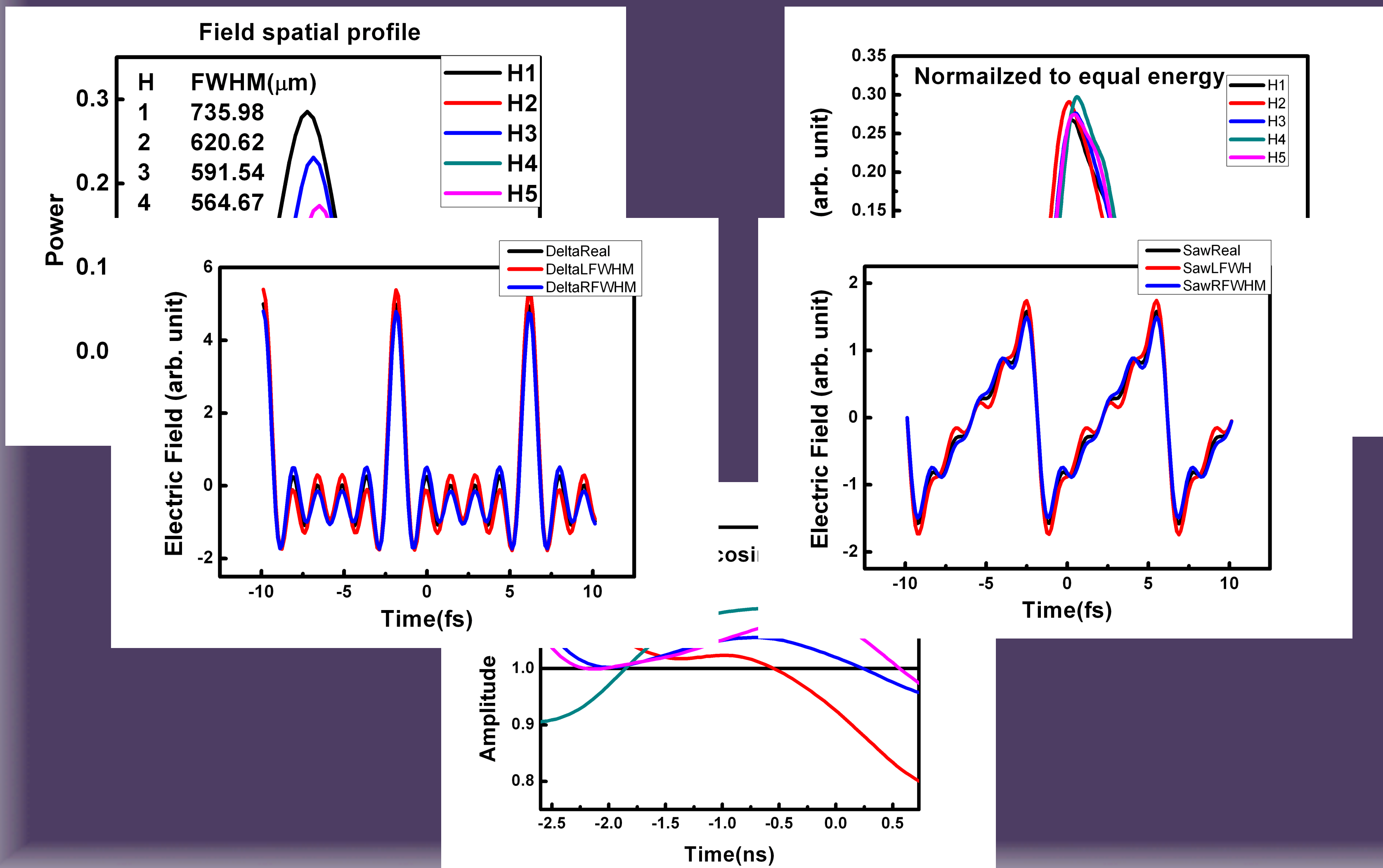
$$E(t) = \sum_n E_n(t) = \sum_n A_n(t)\cos(\omega_n t + \phi_n)$$

$\omega_n = n\omega_m$

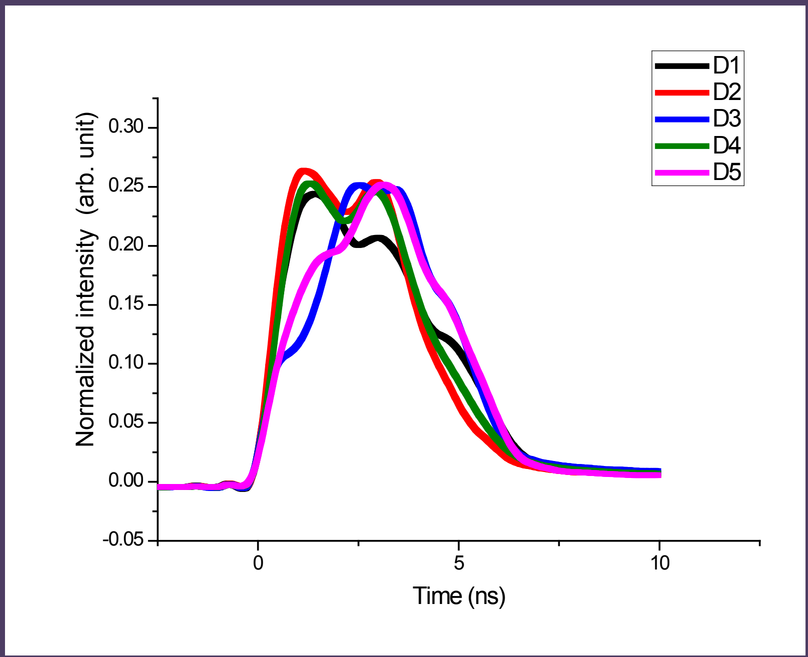
single cycle cosine
saw tooth
square
triangle

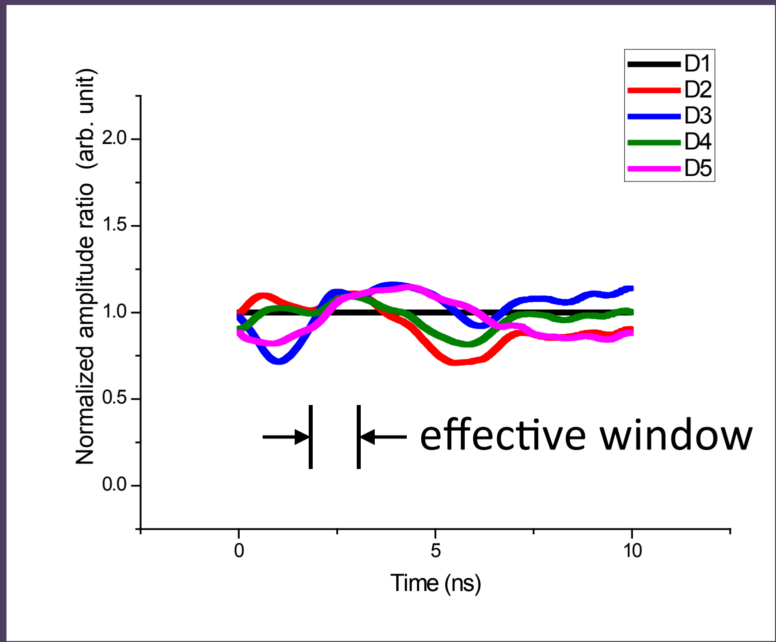


Fourier synthesis is temporal and spatial superposition of harmonic fields



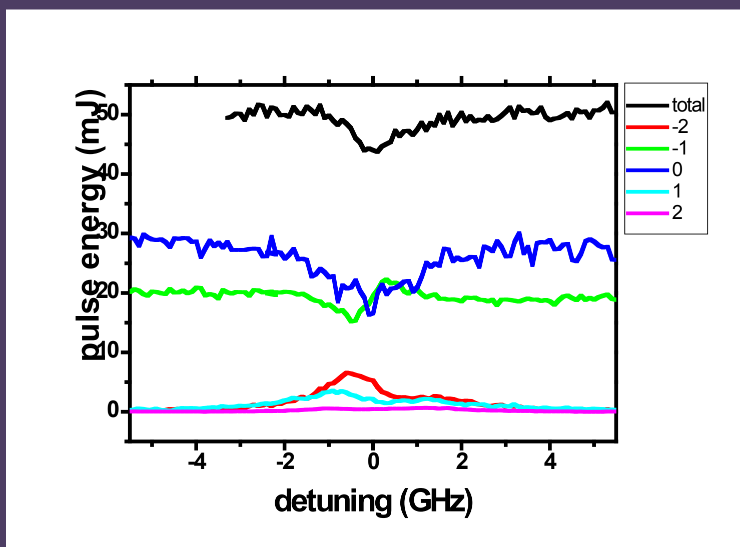
Pump depleted harmonic conversion



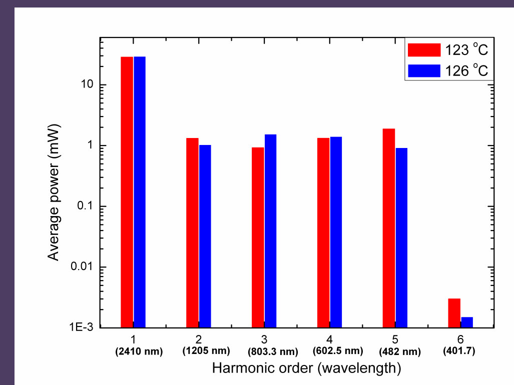


For example, high power lasers

Favors non-depleted pump conversion to each harmonic



Raman



QPM-SFM

Pulse duration: 5 ns
Pulse energy
1064 nm: 380 mJ
532 nm: 178 mJ
355 nm: 70 mJ
266 nm: 41 mJ
213 nm: 22 mJ

Commercial Nd:YAG laser

Is there a better way?
Do we have a remedy ?

Go to extremes:

- Continuous-train ultrashort waveforms
- Isolated single-cycle waveforms

collaborators

Waveform Synthesis

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