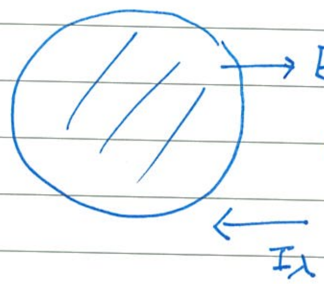
Chapter I. → 之前 $\begin{cases} \text{光} \Rightarrow \text{波動} \\ \text{電子、質子} \Rightarrow \text{粒子} \end{cases}$.

啟開量子力學大門的可說是 Blackbody radiation 之理論工作, so we shall start by considering the Blackbody radiation:

Blackbody radiation

1. Any object with finite temperature radiates.



$\to E_\lambda(T)$ = the emissive power (energy / area · time · for λ)

$\leftarrow I_\lambda$ $\qquad \lambda$ = wavelength.

At same time, it also absorbs radiations.

$A_\lambda I_\lambda$ = energy absorbed / area · time for λ

I_λ: incident energy / area · time

A_λ: absorptivity (fraction of absorbed!)

註: 當物件很厚, $A_\lambda + R_\lambda$ (reflectivity) $= 1$, 沒有 transimission

2. Kirchhoff's law

Equilibrium : $\frac{E_\lambda}{A_\lambda}$ = same for all objects.

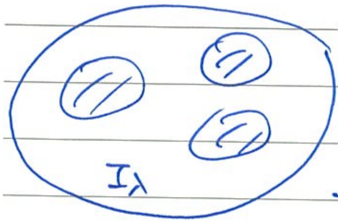
Simple arguement : equilibrium implies

① $E_\lambda = A_\lambda I_\lambda \quad \therefore \frac{E_\lambda}{A_\lambda} = I_\lambda$

(如果不同的話，不會 stationary)

② I_λ is the same for all objects

This is actually a result of a more general statement:



isothermal enclosure

same T

(As we'll see, $I_\lambda \propto$ energy density of radiation)

I_λ : same for different direction & positions.

否則會違反 2nd law!

(see page 1-4 for an example)

3. Black body

① 起源

由以上可知 $\frac{E_\lambda}{A_\lambda} = f(\lambda) =$ 只是T和λ之函數 (universal function)

A_λ depends on the geometry & composition of the object, $\therefore E_\lambda(T)$ is not universal generally!

However, if $A_\lambda = 1$ for all λ

此時 $E_\lambda(T) = f(\lambda)$ is universal

This is the reason why it is interesting to study the case $A_\lambda = 1$ (for all λ)

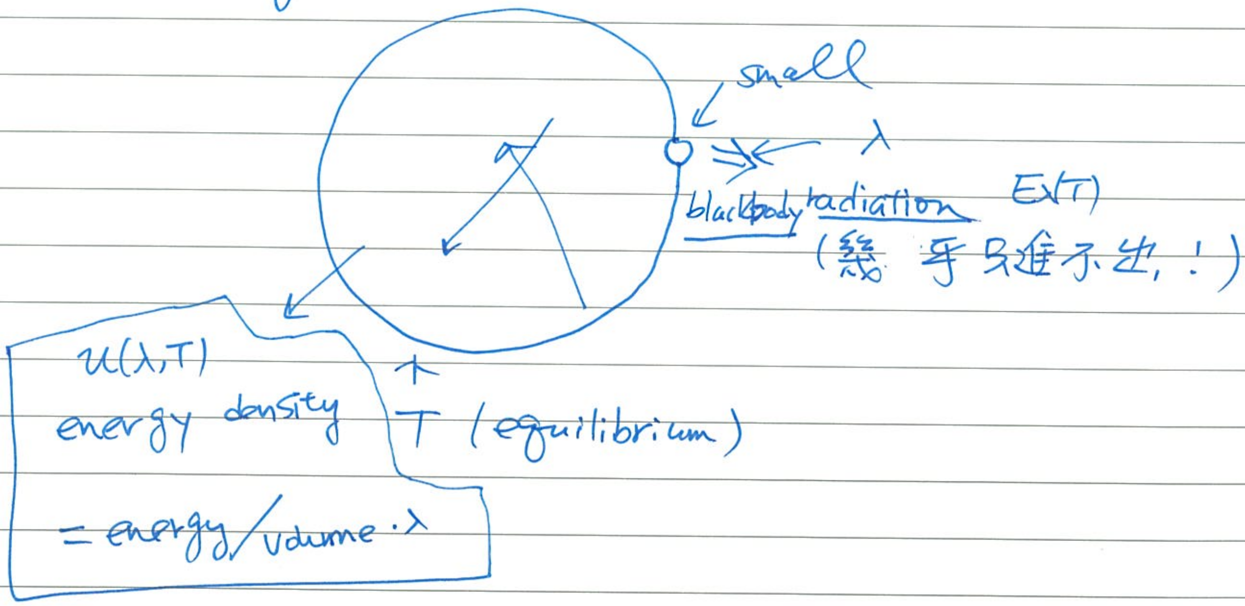
由於 $A_\lambda=1$ for all λ 包括了 可見光 ($\lambda \simeq 400-700\,nm$), so it is black.

We call $E_\lambda(T)=f(\lambda)$ as the black body radiation spectrum!

② How to form a black body?

真實的物體並不會像 black body 一樣 $A_\lambda=1$ for all λ. then how can we speak of the black body?

The following model (cavity) was invented to simulate the black body:



it turns out there is a relation between $E_\lambda(T)$ & $u(\lambda,T)$: $u(\lambda,T)=\dfrac{4E(\lambda,T)}{c}$

∴ Universality of $E_\lambda(T)$ $\Leftrightarrow$ $u(\lambda,T)$ is universal!
(∵ only depends on λ & T 與 geometry 無關)

∵ Our universe turns out to be a big cavity (no material exist) in early times.

⇒ $u(\lambda, T)$ is also "black-body type"!

What is $u(\lambda, T)$?

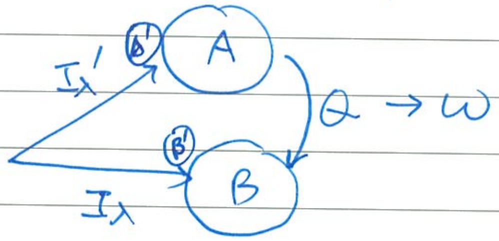
4. Early understandings of $u(\lambda, T)$ for cavity

① isotropic (Kirchhoff)

如果平衡時非 isotropic：

取(A,B)為一樣材料所作之物件置於不同方向.

$I_\lambda' > I_\lambda$



eventually $T_A > T_B$

connecting $A \to B$ can form a Carnot engine to extract work from heat in an equilibrium system

Violate the 2nd law!

② $E(\lambda, T) = \frac{c}{4} u(\lambda, T)$ ← 只是λ與T之函數 與 Geometry 無關! (Kirchhoff's law!)

this is essentially the average version of the relation $\boxed{\vec{J} = \rho \vec{v}}$

如 電流密度 $\vec{J_e} = \rho_e \cdot \vec{v_e}$.

∵ radiation velocity = c

understanding $\vec{j} = \rho v$:

$$Qv = \frac{\text{charge}}{\text{sec}} \cdot m$$

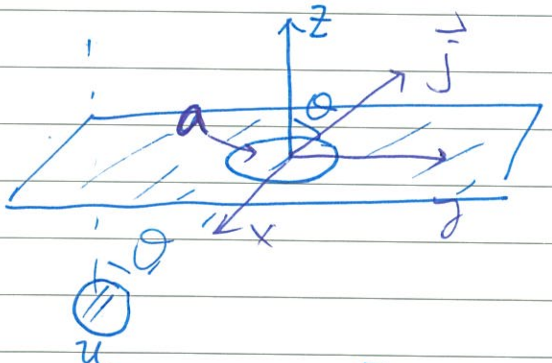
$$\therefore \frac{Qv}{V} = \rho v = \frac{\text{charge}}{\text{Area sec}} = \frac{\text{current}}{\text{Area}} = \frac{I}{A}$$

$$\because I = A j \quad \therefore \; j = \rho v$$

$\Uparrow$ 电流密度 $\quad I = \vec{j} \cdot \vec{A}$

This applies to "other charges" carried by the particles too! $\boxed{\text{The above is for flow with one direction only!}}$

To derive the relation, we choose the following coordinate system:



Only j_z contributes

$(\vec{j} \cdot \vec{A} = I)$

$$\because \vec{j} = u\vec{c} \quad \text{(isotropic)}$$

$$j_z = uc\cos\theta$$

$$\therefore \text{total flux} \equiv E(\lambda, T) = \frac{\int_0^{\pi/2} \sin\theta \, d\theta \int_0^{2\pi} d\phi \, j_z a}{a \cdot \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi}$$

$$= \frac{1}{4\pi} \cdot uc \cdot 2\pi \int_0^{\pi/2} \sin\theta \, d\theta \cos\theta$$

$$= \frac{1}{4} c u$$

除以 4π 是因為 given a volume V of energy density u only $\frac{1}{4\pi} \sin\theta \, d\theta \, d\phi$ 之 fraction can escape to the hole and goes out!

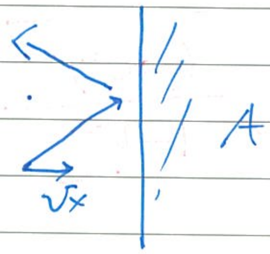
③ equation of state (Boltzman) (for perfect reflecting cavity)

$\downarrow$

∵ universal 可選的較簡單的作

ordinary ideal gas : $PV = nRT = \frac{2}{3}E$

recall :



$$\Delta p = 2mv_x \cdot (v_x A \Delta t) \cdot n(v_x)$$

$$\therefore \text{Pressure} \equiv P = \sum_{v_x > 0} 2m\, v_x^2\, n(v_x)$$

$$= \sum_{v_x} m v_x^2\, n(v_x)$$

$$= \frac{1}{3} m \langle v^2 \rangle n \qquad PV = \frac{2}{3}E$$

light : $m = 0$. $E = pc$ $\sqrt{(m_0c^2)^2 + (pc)^2}$

$\Downarrow$ perfectly reflecting

$$\Delta p = 2p_x \cdot (c_x A \Delta t) \cdot n(c_x)$$

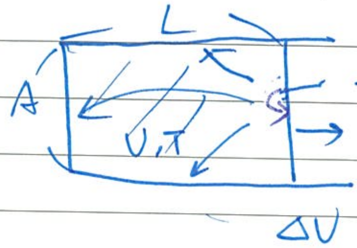
$$P = \sum_{c_x > 0} 2p_x c_x n(c_x)$$

$$= \sum_{c_x} p_x c_x n(c_x)$$

$$= \frac{1}{3} \langle pc \rangle \cdot n$$

$\therefore PV = \frac{E}{3}$, $P = \frac{1}{3}u$ ∴ P 也與 geometry 無關!

④ The Stefan-Boltzman law



perfectly reflecting

expand to $(V+\Delta V,\ T+\Delta T)$

$$u = \int_0^\infty u(\lambda, T)\, d\lambda$$

↳ depends only on T

$$dQ = dU + p\,dV = d(uV) + \frac{u}{3} dV$$

$$\parallel$$

$$TdS \quad = \frac{4}{3} u\, dV + V\, du$$

$$= \frac{4}{3} u\, dV + V \frac{du}{dT} dT$$

$$\therefore \frac{\partial S}{\partial T} = \frac{V}{T}\frac{du}{dT}, \quad \frac{\partial S}{\partial V} = \frac{4}{3}\frac{u}{T}$$

$$\because \frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} \quad \therefore \frac{1}{T}\frac{du}{dT} = \frac{4}{3}\frac{d}{dT}\left(\frac{u}{T}\right)$$

$$\therefore \frac{du}{dT} = \frac{4u}{T} \quad \therefore u \propto T^4$$

i.e. the total emissive power $E(T) = \int_0^\infty E(\lambda, T)\, d\lambda = \sigma T^4$

Since u is universal, $\therefore u \propto T^4$ also applies ↱ 書 Example 1-1

$U = aT^4$. $a = 7.5662 \times 10^{-16}\ J/m^3 \cdot K^4$

to cavities that does not have perfectly reflecting walls.

⑤ Wien's law

The above is for slow expansion such that $u(\lambda, T)$ (the distribution) is fixed!

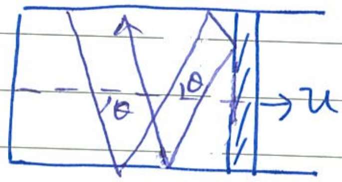
Wien considered the adiabatic expansion. In this case, $\because 0 = dQ = \frac{u}{3} dV + d(uV)$

$V = AL \quad \therefore \frac{du}{u} = -\frac{4}{3}\frac{dL}{L} \quad u \propto L^{-4/3} \propto T^4$

(也適用單色光)

即 u 與 ν 有關！這似乎與 u depends only on T 有矛盾，其實不然，∵ T 可以與 ν 有關，特別是在 adiabatic 之過程 ν 與 T 並不完全獨立！

Wien 注意到當 piston 以速度 u 向右移時，反射之 radiation 有 Doppler shift



$$\frac{\Delta\lambda}{\lambda} = \frac{2u}{c}\lambda\cos\theta \quad —①$$

[Ex. $\lambda_1 = \lambda\sqrt{\frac{1+u/c\cos\theta}{1-u/c\cos\theta}}$ = piston 所看到之波長　$f=f_0\sqrt{\frac{1+\beta}{1-\beta}}$ approaching

$\simeq \lambda(1+\frac{u}{c}\cos\theta)$

piston 再發出之 $\lambda' = \lambda_1\sqrt{\frac{1+u/c\cos\theta}{1-u/c\cos\theta}} \simeq \lambda(1+2\frac{u}{c}\cos\theta)$]

這個 Doppler shift 會重新分配 $u(\lambda,T)$！

在導 Stefan-Boltzman law 時，外界之熱量可以流進 而使 ∴ $u(\lambda,T)$ $(u(T)) \rightarrow u(T+\Delta T)$！

但在 adiabatic 時，∵ wall is perfectly reflecting，外界沒熱量流入，radiation 必須自我調整以降低溫度 T！

這個調整正好是透過 Doppler shift 達成：

波長在 λ 附近之 radiation 每 $\frac{2L}{c\cos\theta}$ 秒歷經一次
(連續 $\angle\theta$)

Doppler shift　∴ $\frac{\Delta\lambda}{\Delta t} = \frac{c\cos\theta}{2L}\cdot\frac{2u}{c}\lambda\cos\theta$

$\because u\Delta t = \Delta L$

$$\therefore \frac{\Delta\lambda}{\lambda} = \frac{\Delta L}{L}\cdot\cos^2\theta$$

Now, if we replace the smooth surface by rough surface, then each time, the radiation hits the wall with different angles

$\cos^2\theta \to \langle\cos^2\theta\rangle = \frac{1}{3}$

或是 argue：given λ，各個方向之机率一樣

$\langle\cos^2\theta\rangle = \frac{1}{3}$

$$\frac{\int_0^\pi \cos^2\theta \, d\cos\theta}{\int_0^\pi d\cos\theta} = \frac{1}{3}$$

$$\therefore \frac{\Delta\lambda}{\lambda} = \frac{\Delta L}{3L} = -\frac{1}{3}\frac{3}{4}\frac{\Delta u}{u} = -\frac{\Delta T}{T}$$

$u \propto T^4$

即 $\ln\frac{\lambda_2}{\lambda_1} = \ln\frac{T_1}{T_2}$

如果在 adiabatically 过程中 維持平衡

換句話說，在 adiabatic 过程中，原先之 λ_1 的 radiation 會跑到 λ_2 去，

$$\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2}$$

T_1 ← 原先之溫度

T_2 ← 最後之 〃 〃

則原先之区域 $(\lambda_1, \lambda_1+d\lambda_1) \to (\lambda_2, \lambda_2+d\lambda_2)$

$$\frac{\lambda_2+d\lambda_2}{\lambda_1+d\lambda_1} = \frac{T_1}{T_2} \quad \text{is also satisfied}$$

$$\therefore \frac{d\lambda_2}{d\lambda_1} = \frac{T_1}{T_2} \quad \text{too!}$$

energy density / wavelength

$\therefore$ 区間之大小會變化，並影響了 $u(\lambda, T)$：

$$\frac{u(\lambda_2, T_2)\,d\lambda_2}{u(\lambda, T_1)\,d\lambda_1} = \frac{u_2}{u_1} = \left(\frac{T_2}{T_1}\right)^4$$

而 $\lambda \propto L^{\frac{1}{3}} \propto T^{-1}$ ← 想像推 $u \propto L^{-4/3}$ 可以只有單色光在 cavity 中，此式也成立

$$\therefore \frac{u(\lambda_2, T_2)}{u(\lambda_1, T_1)} = \left(\frac{T_2}{T_1}\right)^5$$

即 $$\frac{u(\lambda_2 T_2)}{T_2^5} = \frac{u(\lambda_1 T_1)}{T_1^5} = 常数$$

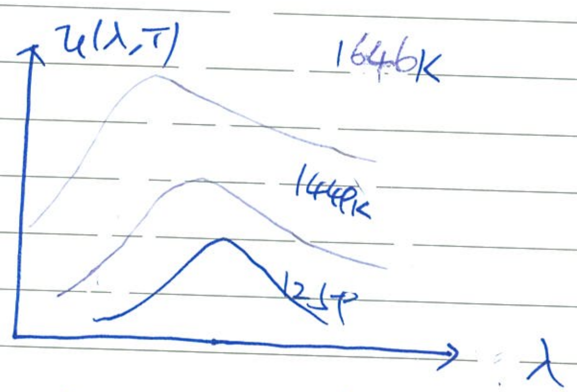
此常数 independent of any λ & T that satisfy $\lambda T =$ 常数 $\left(\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2}\right)$

$$\therefore u(\lambda, T) = T^5 \cdot F(\lambda T)$$

$$= \frac{(\lambda T)^5}{\lambda^5} F(\lambda T) = \frac{f(\lambda T)}{\lambda^5}$$ ↙ universal function

discussions:

(i) implication: Collapse of data



collapse

this trick has been used very often in Condensed matter expt. nowadays!

$\boxed{\text{Know } f \Rightarrow \text{any } u(\lambda, T) \text{ is Known!}}$

(ii) λT is not dimensionless! to make it dimensionless, we need other constants such as c, k_B, ... you will see it later! right now you may try to construct it

$\rightarrow \frac{K_B}{hc}$

$\lambda T \cdot \underset{\Uparrow}{\overset{\times}{const}} = \text{dimensionless}$

$[K_B] = \frac{\text{Energy}}{K}$ $\quad Kg \cdot \frac{L^2}{T^2}$

$[\quad] = \frac{1}{L \quad K} = \frac{[K_B]}{L \cdot \text{Energy}}$ $\quad [C] = \frac{L}{T}$

$= \frac{[K_B]}{L \cdot Kg \cdot c^2} = ?$ $\quad u(\lambda, T) = \frac{\bigcirc}{\lambda^5} f(\lambda T)$ ←常 const.

(iii) max : $\lambda_{max} T =$ 常数

$\therefore \lambda_{max} = \frac{b}{T}$ (Wien's law)

$b = 0.2898$ cm-K (universal)

(iv) It is more convenient to discuss $u(\nu, T)$

$u(\nu, T) |d\nu| = u(\lambda, T) |d\lambda|$

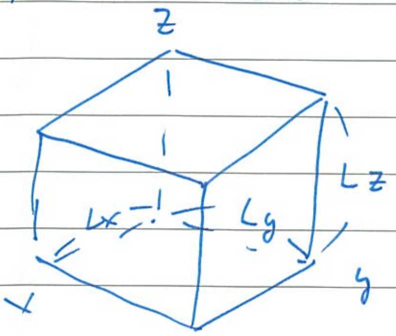
$\therefore u(\nu, T) = u(\lambda, T) \left|\frac{d\lambda}{d\nu}\right| = \frac{c}{\nu^2} u(\lambda, T) = \frac{c}{\nu^2} \cdot \frac{1}{\lambda^5} f(\lambda T)$

$\uparrow$ $\lambda = \frac{c}{\nu}$ $\quad \equiv \nu^3 g\left(\frac{\nu}{T}\right)$

$\frac{\nu}{T} \cdot y = \text{dimensionless}$ $\quad cx = y$, $u(\lambda, T) = \frac{c}{\lambda^2} u\left(\frac{c}{\lambda}, T\right)$

⑥ Rayleigh - Jeans law ⇒ $\because u$ is universal, 所以可以取正立方體, 且用 periodic b.c.

(i) # of modes plane waves



$e^{i\vec{k}\cdot\vec{r}}$ (見 Chapt. 2-1, 2-2)

periodic B.C.

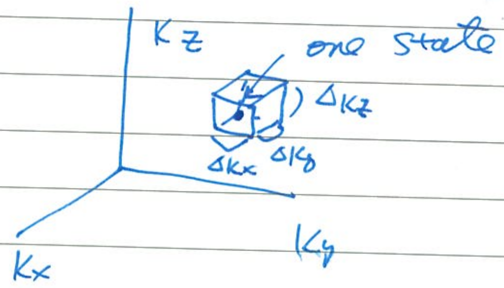
$k_x L_x = 2\pi n_x$

$k_x = \frac{2\pi}{L_x} n_x$, $n_x = 0, \pm 1, \pm 2, \ldots$

$\Delta k_x = \frac{2\pi}{L_x}$，即每 Δk_x 即有一 state

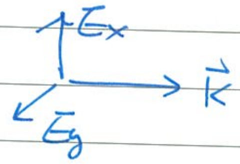
Similarly, $k_y = \frac{2\pi}{L_y} n_y$, $k_z = \frac{2\pi}{L_z} n_z$

∴在 k-space 中，每 $\Delta^3 k = \frac{2\pi}{L_x} \frac{2\pi}{L_y} \frac{2\pi}{L_z}$ 有一 state



$$= \frac{(2\pi)^3}{V}$$

不過 each $\vec{k}$ 可以有 2 個 polarization



∴ 在k空間中，每 $\frac{(2\pi)^3}{V}$ 有 2 個 state

∴在 $|\vec{k}|=k$ 與 $|\vec{k}|=k+dk$ 中間之球殼中之

$$\text{state 數} = \frac{4\pi k^2 \cdot dk}{(2\pi)^3/V} \times 2$$

$$= \frac{V k^2 dk}{\pi^2}$$

∵ $\omega = ck$, $\omega = 2\pi\nu$

$$\therefore = \frac{V}{\pi^2}\left(\frac{2\pi}{c}\nu\right)^2 \frac{2\pi}{c} d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu$$

$d\sigma$ 為 cavity 中 mode 在 $(\nu, \nu+d\nu)$ 之間的數目！

$\dot{E}$

SHO 平均動能 $\frac{1}{2}k_BT$
平均位能 $\frac{1}{2}k_BT$ E

(ii) Rayleigh 及 Jeans 認為

每一個 mode 之平均能量 $\bar{\varepsilon} = k_BT$ (equipartition law)

$$\therefore \; u(\nu,T)\,d\nu = \frac{k_BT \cdot \frac{8\pi V}{c^3}\nu^2 d\nu}{V}$$

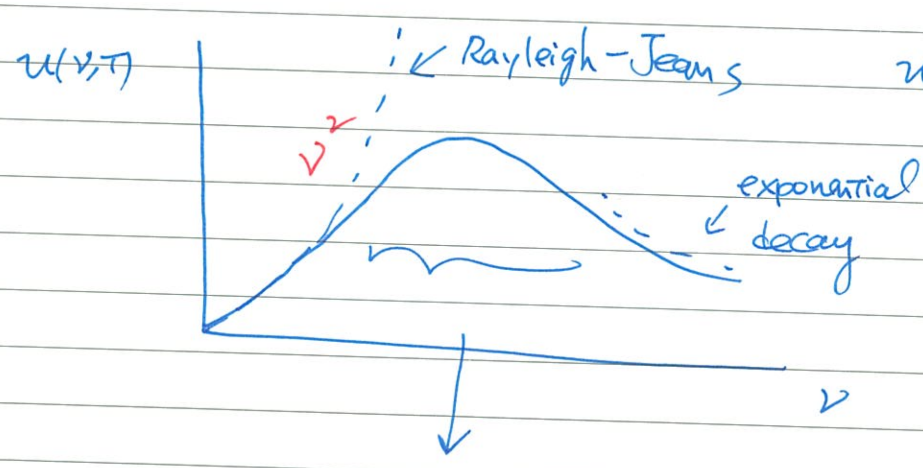
$\Uparrow$

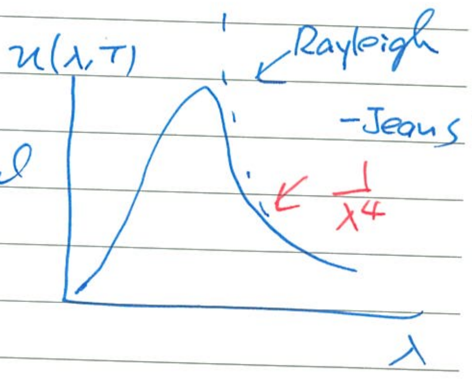
energy density / volume per ν.

基本上：球面積 $\propto \nu^2$.

$$\Rightarrow \; \therefore u(\nu,T) = \frac{8\pi \nu^2 k_BT}{c^3} \quad \Rightarrow \quad u(\lambda,T) = \frac{c}{\lambda^2}u(\nu=\tfrac{c}{\lambda},T) = \frac{8\pi k_BT}{\lambda^4}$$

Rayleigh - Jeans formula.





ultraviolet catastrophe

Historical:
Wien: large ν
$u \sim e^{-\frac{\beta\nu}{k_BT}}$

as a result, $\underline{\int_0^\infty u(\nu,T)\,d\nu = \infty}$

顯然不对

5. The Planck distribution

從以上所討論可知 $u(\nu,T) = \bar{\varepsilon} \cdot \frac{8\pi\nu^2}{c^3}$

所以問題出在 $\bar{\varepsilon} = k_BT$!

Planck 原先的 approach 是先找到一個好的

interpolation:

$$u(\nu,T) \to 0 \qquad \nu \to 0$$

$$\to e^{-\beta\nu/k_BT} \qquad \nu \to \infty$$

interpolate: $u(\nu,T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT}-1}$ $\quad h = 6.63\times10^{-27}$ erg sec

Planck constant.

effectively, 引入 $h\nu$ as cut off :

當 $h\nu \ll k_BT$. $\quad e^{h\nu/k_BT}-1 \simeq h\nu/k_BT$

$$u(\nu,T) \sim \frac{8\pi h}{c^3}\frac{\nu^3}{h\nu/k_BT} \sim \frac{8\pi\nu^2}{c^3}k_BT$$

recover Rayleigh-Jeans form

$h\nu \gg k_BT$, $\quad u(\nu,T) \sim \frac{8\pi h}{c^3}\nu^3 e^{-h\nu/k_BT}$

而 Wien's form 一樣.

之後, 他想到, 這個 interpolation 事實上在說

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/k_BT}-1}$$

, 換句話說 equipartition law 應該修正!

但這個 form 代表什麼意思呢?

先由推導來看，那裏可能出錯：

古典：

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon e^{-\varepsilon/k_BT} d\varepsilon}{\int_0^\infty e^{-\varepsilon/k_BT} d\varepsilon}$$

(注意：如果改算 E_k，

$$\bar{\varepsilon} = \frac{\int_{-\infty}^{\infty} \frac{p^2}{2m} e^{-\beta\frac{p^2}{2m}} dp}{\int_{-\infty}^{\infty} e^{-\beta p^2/2m} dp} = \frac{1}{2}k_BT\ !)$$

$$\int_0^\infty e^{-\varepsilon/k_BT} d\varepsilon = k_BT \int_0^\infty e^{-x} dx = k_BT$$

$$\int_0^\infty \varepsilon e^{-\varepsilon/k_BT} d\varepsilon = (k_BT)^2 \int_0^\infty x e^{-x} dx = (k_BT)^2$$

$$\therefore \bar{\varepsilon} = k_BT$$

Planck : to produce $\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/k_BT}-1}$

assume ε is not continuous but only takes the values at $nh\nu$ $n=0,1,2,3\ldots$

$$\therefore \bar{\varepsilon} = \frac{\sum_{n=0}^\infty n\varepsilon e^{-n\varepsilon/k_BT}}{\sum_{n=0}^\infty e^{-n\varepsilon/k_BT}} \qquad \varepsilon = h\nu$$

$$= \frac{-\epsilon \frac{d}{dx}\sum_{n=0}^\infty e^{-nx}}{\sum_{n=0}^\infty e^{-nx}} \Bigg|\ x = \varepsilon/k_BT$$

$$= -\epsilon \frac{d}{dx} \ln \sum_{n=0}^\infty e^{-nx} \Bigg|\ x = \varepsilon/k_BT$$

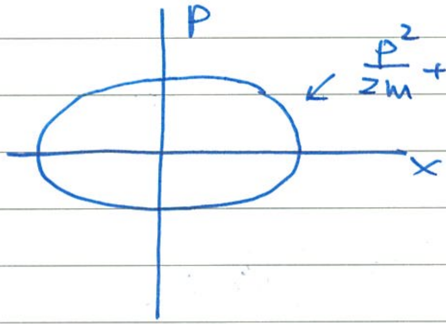
$$\sum_{n=0}^\infty e^{-nx} = \frac{1}{1-e^{-x}} \qquad \frac{d}{dx}\ln(1-e^{-x}) = \frac{e^{-x}}{1-e^{-x}}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

Harmonic oscillator

$$\varepsilon = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\bar{\varepsilon} = \frac{\int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dx \; e^{-\beta\varepsilon} \cdot \varepsilon}{\int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dx \; e^{-\beta\varepsilon}}$$



$$\frac{p^2}{2m} + \frac{1}{2}kx^2 = \varepsilon$$

$$p = \sqrt{2m}\, y$$
$$x = \sqrt{\frac{2}{k}}\, x$$

$$x^2 + y^2 = r^2 = \varepsilon$$
$$r = \sqrt{\varepsilon}$$

$$\int dp \int dx = \sqrt{2m}\sqrt{\frac{2}{k}} \int dy \int dx$$
$$= \sqrt{2m}\sqrt{\frac{2}{k}} \cdot 2\pi \int_0^{\infty} r\,dr$$
$$= \sqrt{2m}\sqrt{\frac{2}{k}} \cdot \pi \int_0^{\infty} d\varepsilon$$

$$\therefore \; \bar{\varepsilon} = \frac{\int_0^{\infty} d\varepsilon \; \varepsilon e^{-\beta\varepsilon}}{\int_0^{\infty} d\varepsilon \; e^{-\beta\varepsilon}}$$

如此

$$\bar{\varepsilon} = \frac{\varepsilon}{e^{\varepsilon/k_BT}-1} = \frac{h\nu}{e^{h\nu/k_BT}-1}$$

is obtained!

∴ Planck: 假設, for some unknown reason,

牆上之原子放出及吸收的 radiation 只能以 $nh\nu$ 進行

"quanta" (energy $= nh\nu$)

(i) Reproducing Wien's law

$$u(\nu,T) = \nu^3 \cdot \left(\frac{8\pi h/c^3}{e^{h\nu/k_BT}-1}\right)$$

$$\Uparrow$$

$$g\left(\frac{\nu}{T}\right), \quad g(x) = \frac{8\pi h/c^3}{e^{\frac{h}{k_B}x}-1}$$

$$\Uparrow$$

to make it dimension

$$y = \frac{h}{k_B}$$

$$\tilde{u}(\lambda,T) = \frac{\nu^2}{c}u(\nu,T)$$

不能用同一 notation ∵函數不同

$$= \frac{c}{\lambda^2}\cdot\frac{8\pi h}{c^3}\frac{(c/\lambda)^3}{e^{\frac{hc}{\lambda k_BT}}-1}$$

$$= \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda k_BT}-1}$$

(ii) Reproducing Stefan-Boltzmann law

$$\int_0^\infty u(\nu,T)\,d\nu = \frac{8\pi h}{c^3}\int_0^\infty d\nu \frac{\nu^3}{e^{h\nu/k_BT}-1}$$

$$= \frac{8\pi h}{c^3}\left(\frac{k_BT}{h}\right)^4 \int_0^\infty \frac{x^3\,dx}{e^x-1}$$

$\frac{\pi^4}{15}$ (exercise)

$$= aT^4, \quad a = 7.56\times10^{-15}\ \text{erg}/\text{cm}^3\text{K}^4$$

6. Other relevant examples

(i) Failure of Dulong - Petit law

Dulong - Petit law :

Solid : each degree of freedom

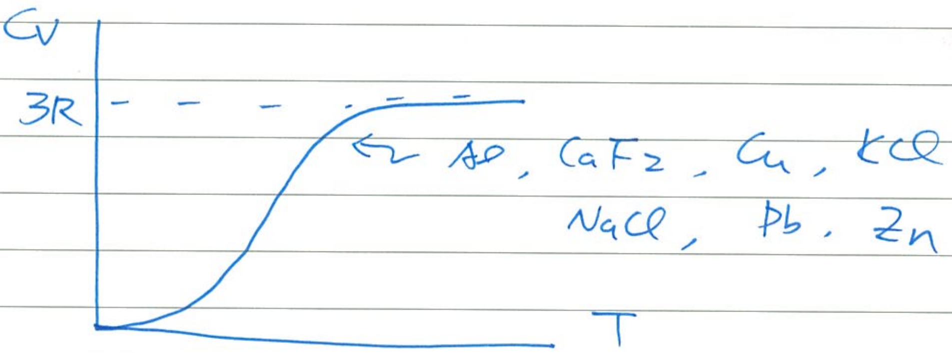
$\Rightarrow$ energy $= k_B T$

N atoms

$\therefore 3Nk_BT = E = 3RT$

$\therefore C_v = \frac{dE}{dT} = 3R = const.$

expt.



correct: 將 Phonon (elastic wave) 視為光 (電磁波) 一樣, $\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/k_BT} - 1}$ too. ν = frequency of elastic wave.

(ii) The Cosmic Microwave Radiation Background (CMRB)

Penzias & Wilson (1964) discovered that there is a background of radiation in our surroundings. $\rightarrow$ 叫做 background cosmic microwave radiation

(∵ λ在 microwave 附近) 見圖 I-3

① 与 black body 一樣之 spectrum！
$T \simeq 2.735K$ $\lambda_{max} T = 2.898 \times 10^{-3}\ m°K$

$\lambda_{max} \simeq 10^{-3} m = mm$.
(當時量的結果不準，$T \sim 3.5K$
且只量一個 $\lambda = 7.35cm$)

一般相信 我們的宇宙目前對 radiation 基本上
是透明的，且 radiation 不互相達到 thermal equilibrium
則 radiation 以為 很久以前 物質與 radiation
達熱平衡之殘存的 radiation (即大爆炸之殘餘)
↳ confirm 大爆炸理論之 scenario

big-bang model
事實上，根據大爆炸理論，宇宙開始之溫度
極高，那時物質與 radiation 作用（散射 by electrons）
很強，達到熱平衡，一直到 3000°K 時 e 與質子(neucleon)
(約300,000年)
⇒ 之後 (P.J.E. Peebles) 光就基本上與物質 decouple.
原子
隨著宇宙 expand，光之 temperature ↓ 至今.

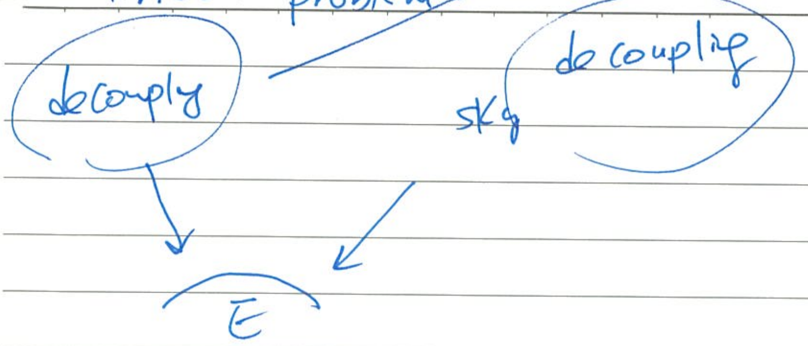
② isotropic！ 扣掉各方向上可能所受 galaxy 之
影響後，CMRB 有很高之
isotropy (均向性)！

(E)

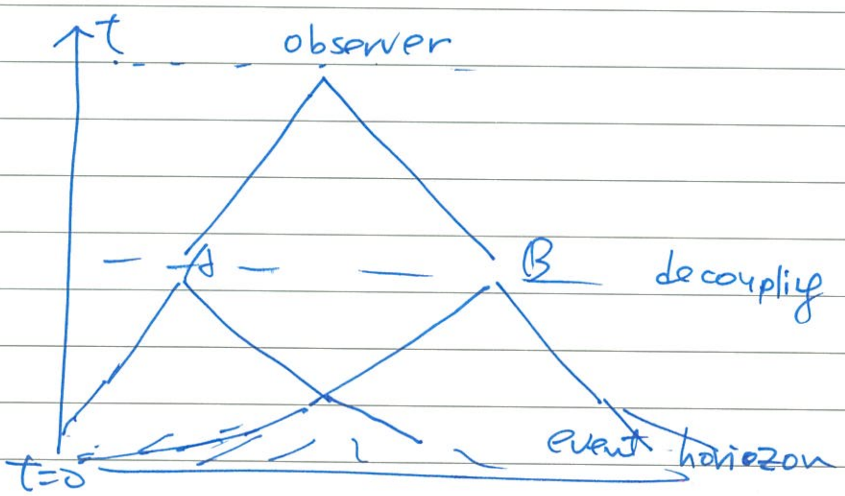
$$\frac{\Delta T}{T} \sim 10^{-5}$$

但是想一想，這有很大的問題：(Fig 1.4)

Horizon problem: → why the same T?



A. Guth (MIT) 在 1981 假設 big bang 之後宇宙曾經經過一個非常快 (exponential growth) expand 的過程 (inflationary)

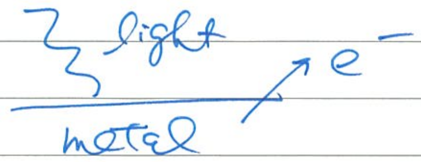


decoupling (last scattering)
horizon of universe
~300 million light years

*最近的測量開始可測到還是有小的 anisotropy (ΔT (在 μK order)) 這些是由於所謂的 quantum fluctuation, 以造成日後之各 galaxy ... 許多團隊 (Taiwan, AMIBA) 都在作更精密之測量, 見 Physics World, Volume 15, No.8, Page 21, 2002

Photoelectric Effect (光電效應)

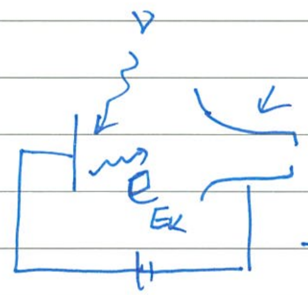
1. 1887 Hertz



此效應事實上可以用來偵測電子能量與 $\vec{p}$ 之關係 (dispersion relation)

↳ 凝體物理: Angle Resolved Photoemission (ARPES)

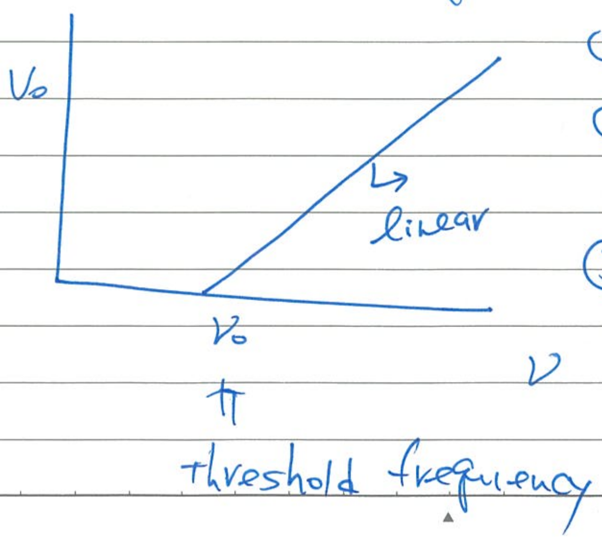
2. expt. findings.



-ve voltage to stop electrons' flow

⇒ stopping potential V_0 (retarding), $eV_0 = (E_K)_{max}$

Millikan



① $\nu < \nu_0$, no e^- comes out

② I (current) $\propto$ intensity of light for $\nu > \nu_0$

③ E_K independent of intensity of light difficulties with classical E.M. (wave) theory:

① intensity $\propto \langle E^2_{(t)} \rangle \propto |\vec{E}|^2$

$\therefore$ intensity $\uparrow$ $|\vec{E}|\uparrow$

⓪ $e^- \to \vec{E}\cos\omega t$, $\vec{E}\uparrow \Rightarrow$ 總是可以把电子 kick out (if intensity is strong enough!)

② intensity $\propto |\vec{E}|$ $\quad e\vec{E} = \text{force}$

$\therefore$ E_k should $\uparrow$ as intensity $\uparrow$

③ time lag (for classical picture, there must be a time delay, but in real expt $\Rightarrow$ no

to shake e^- out, there should be time delay)

a time lag:

example: light. typical 每一Pt原子受到 2.5×10^{-21} joul/sec

游離位能 (Pt) $2.1\,eV = 3.4\times10^{-19}$ Joul

$$\Delta t = \frac{3.4\times10^{-19}\ \text{joul}}{2.5\times10^{-21}\ \text{joul/sec}} = 1.4\times10^{2}\ \text{sec}$$

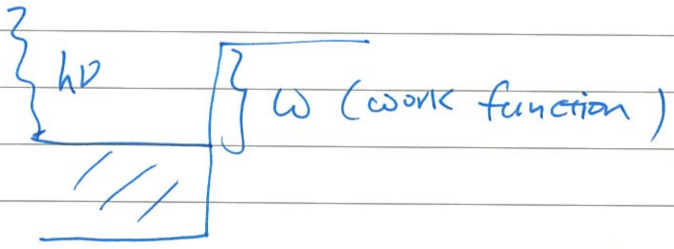
1-22.

3. Einstein's theory

<u>essentially</u>

Photon is particle-like, carry $h\nu$ quanta of energy

photon → absorbed by e^- → emit



metal

$$\frac{1}{2}mv^2 = h\nu - W.$$

$$\therefore \nu_0 = \frac{W}{h} \quad \text{independent of intensity}$$

$$\because \frac{1}{2}mv^2 = eV_0$$

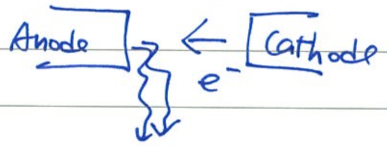
$$\therefore V_0 = \frac{h}{e}\nu - \frac{W}{e} \quad \text{linear in } \nu$$

intensity $\leftrightarrow$ # of photons $\therefore$ intensity $\uparrow$ current $\uparrow$

X rays 的產生 & Spectrum

1. 用以作 Compton scattering 為 X-ray 為 W.K. Roentgen 於 1895年發現的.

其產生 Scheme 如圖所示



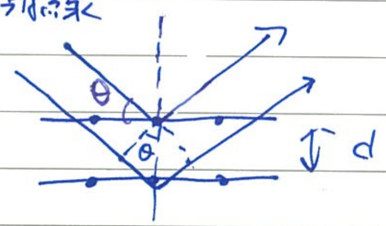
早期对它不太了解, 所以稱為 X-ray

X-ray <
- all materials were trasparent to some degree
- not deflect by magnetic fields

猜測是电子急減速時所放出來的电磁波.

2. Laue 推測 $\lambda \sim 1Å$, 必須以 crystal 中之 atom 來作 grating 才能產生繞射

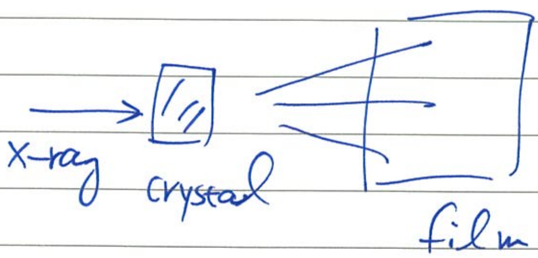
不久就被証實, 在1912時 Bragg 提出一個簡單分析 X射線繞射的方法



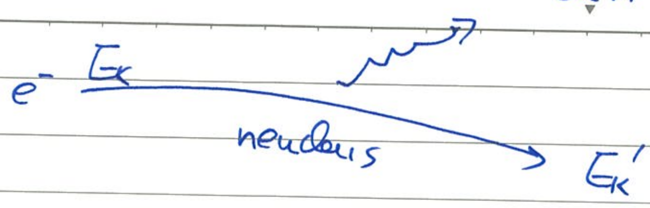
$$2d\sin\theta = m\lambda \qquad m = \text{integers}$$

(Bragg Condition)

Laue's expt



3. Brem sstrahlung photon



$\underbrace{\text{Bremsstrahlung}}$ process

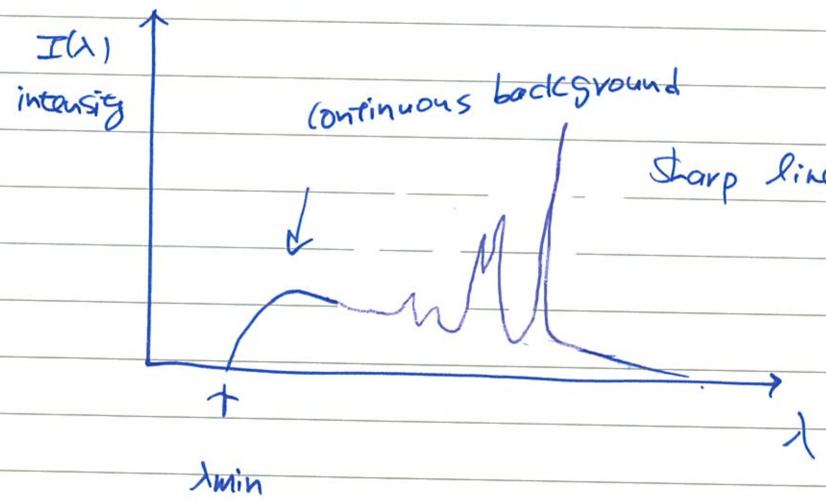
德文 $\begin{cases} \text{Brems} = \text{braking} \\ \text{Strahlung} = \text{radiation} \end{cases}$

$$E_K - E_K' = h\nu = \frac{hc}{\lambda}.$$

$$\therefore \nu_{max} = \frac{E_K}{h}, \quad \frac{hc}{\lambda_{min}} = E_K = eV$$

$$\therefore \lambda_{min} = \frac{hc}{eV}$$

$\therefore$ X-ray spectrum

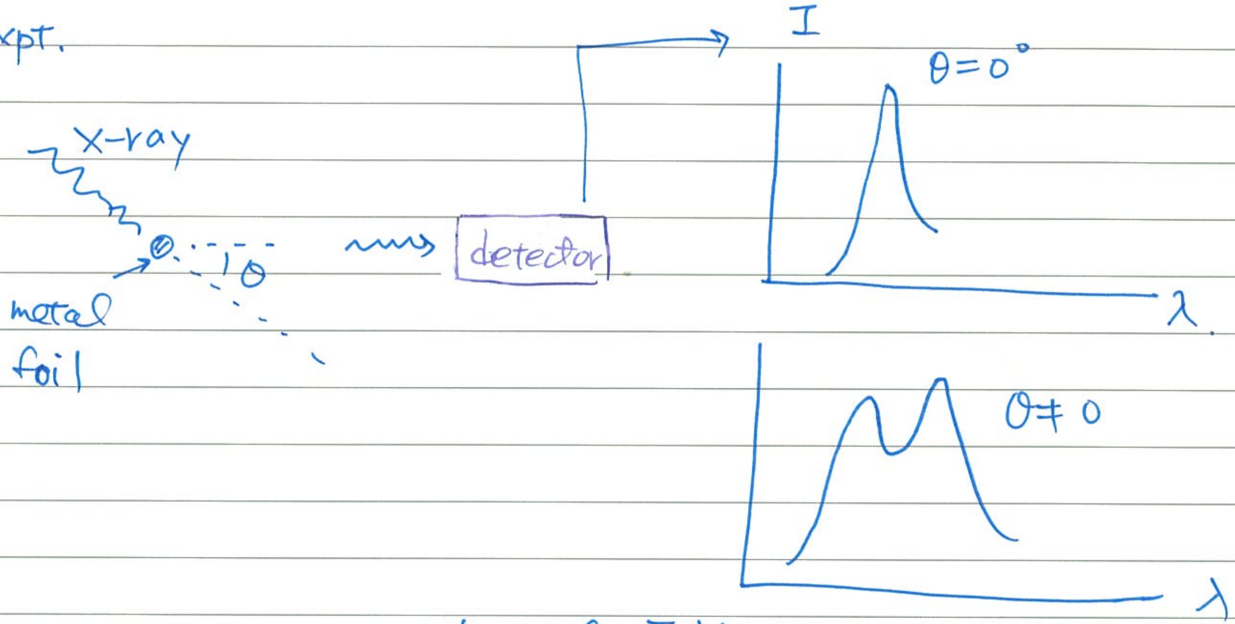


Sharp lines 与 target 之 材料有關 will be explained later.

The Compton Effect

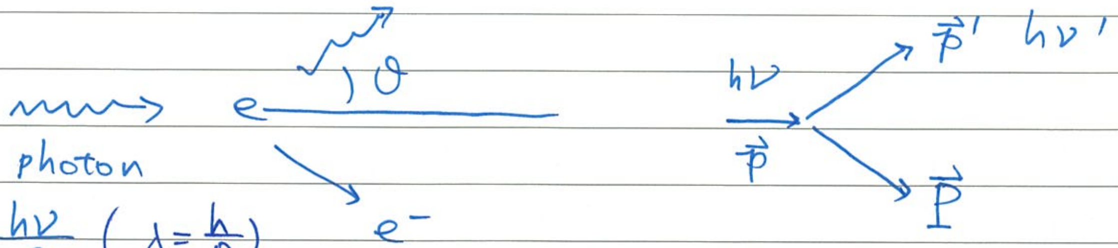
康卜頓效應可說是說明光之粒子性最直接的証據.

1. Expt.



hard to explain by classical E.M. wave Theory!

2. Compton treated light as particles & elastically scattering from e^- :



$p=\frac{h\nu}{c}$ $\left(\lambda=\frac{h}{p}\right)$

⇑ by analogy to relativistic particles

$$\boxed{\begin{array}{l} E=\sqrt{(m_0c^2)^2+(pc)^2}, \quad v=\frac{dE}{dp}=\frac{pc^2}{E}=c \\ \because m_0=0 \\ \therefore E=pc=h\nu \qquad \text{可直接derive } \frac{pc^2}{E}=\frac{\frac{m_0v}{\sqrt{1-\beta^2}}c^2}{\frac{m_0c^2}{\sqrt{1-\beta^2}}} \end{array}}$$

Momentum conservation

$$\vec{p} = \vec{p}\,' + \vec{P}$$

$$\therefore P^2 = (\vec{p}-\vec{p}\,')^2 = p^2 + p'^2 - 2\vec{p}\cdot\vec{p}\,' \quad \text{— ①}$$

Energy conservation

$$h\nu + mc^2 = h\nu' + (m^2c^4 + P^2c^2)^{\frac{1}{2}} \quad \text{— ②}$$

↑ rest mass of electron

$$\because p^2 = \left(\frac{h\nu}{c}\right)^2 \qquad p'^2 = \left(\frac{h\nu'}{c}\right)^2$$

$$\therefore \text{①} \Rightarrow P^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\theta$$

$$P^2c^2 = (h\nu - h\nu')^2 + 2h\nu\, h\nu'(1-\cos\theta)$$

$$\text{②} \Rightarrow \cancel{m^2c^4} + P^2c^2 = (h\nu - h\nu' + mc^2)^2$$

$$= (h\nu - h\nu')^2 + 2mc^2(h\nu - h\nu') + \cancel{m^2c^4}$$

$$\therefore 2h\nu\, h\nu'(1-\cos\theta) = 2mc^2(h\nu - h\nu')$$

$$\therefore \frac{h}{mc}(1-\cos\theta) = c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right)$$

$$= \lambda' - \lambda$$

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

Compton wavelength $\cong 2.4 \times 10^{-10}$ m
$= 0.0243$ Å

除了 $\Delta\lambda$ 外，電子的 recoil momentum

$$p^2c^2 = h^2\left[(\nu^2+\nu'^2) - 2\nu\nu'\cos\theta\right]$$

也可檢驗

另外，也可 檢驗所測到之光子和電子之 recoil 是否同時發生.

這些都指向光具有粒子性！

但我們早已知光具有波動性：如双狹縫干涉

所以光似乎同時具有粒子和波動性！(dual nature)

We'll come back to discuss this point later.

在 1923, De Broglie observed this. 他大膽的提出平常我所知具有粒子性的電子

⇒ 也應有波動性. (matter wave, 物質波)

He suggested $\lambda = \dfrac{h}{p}$

Wave properties of particles

首先，我們看々 ~~這個如果是對的~~

一般的 $\lambda = ?$ baseball ($v = 10\,m/sec$, $36\,Km/hour$)

$$\lambda \cong \frac{6.6\times10^{-34}\,\text{joule-sec}}{\underbrace{1.0\,kg}_{m}\times 10\,m/sec} \cong 6.6\times10^{-25}\,\text{Å} \; !$$

very small!

electron $(\overset{E_k}{\overset{\shortparallel}{100\,eV}})$ $\quad m_e c^2 = 0.511$ MeV ≈ 500 keV

$p = \sqrt{2mE_k}$

$$\lambda = \frac{6.6\times10^{-34}\ \text{joule-sec}}{[2\times\underbrace{9.1\times10^{-34}\,kg}_{m_e}\times100\times\underbrace{1.6\times10^{-19}}_{1eV=1.6\times10^{-19}\,Joule}]^{1/2}}$$

$= 1.2\,\mathring{A}$

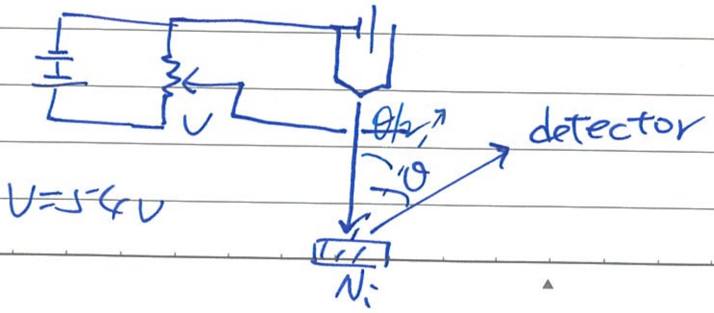
增加 E_k, reduce λ!

問題是如何偵測its波動?

顯然的,要測波動性,必須要讓它產生干涉或繞射的現象!

不管那一種要造成這些現象,其 grating 的 spacing $\sim \lambda$.

歷史上,Davisson 與 Germer 找 Nickel 之 crystal 來作 grating (它的 spacing 才會 $\sim\lambda$) 而說明了電子之波動性 (electron diffraction)



(non-relativistic electrons)

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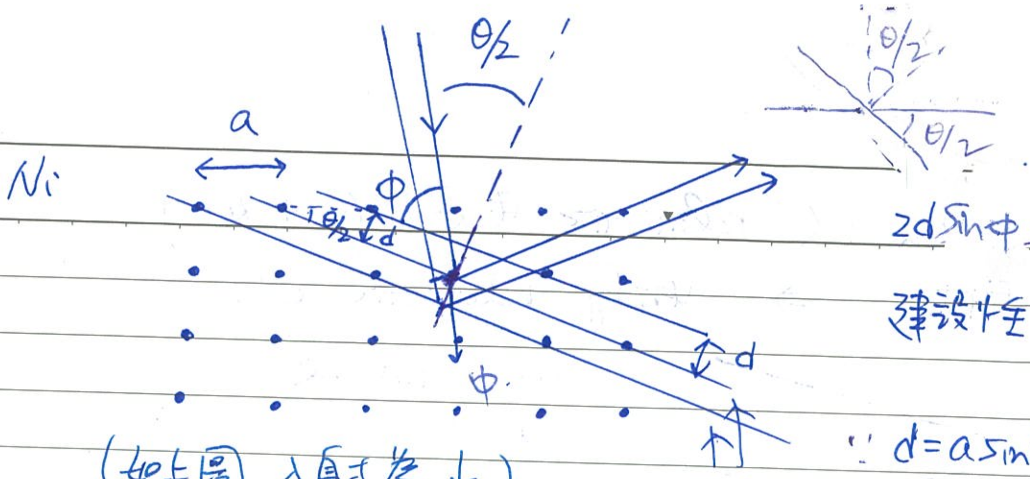


$2d\sin\phi = n\lambda$ 時

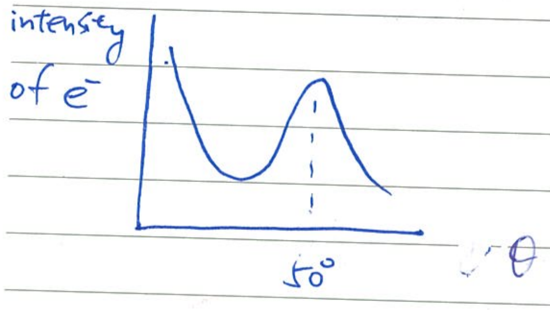
建設性干涉, $\phi = \pi/2 - \theta/2$

$\because d = a\sin\theta/2$, $a = 2.15\,A^\circ$ $\rightarrow 2a\sin\theta/2\cos\theta/2 = n\lambda$ $\rightarrow a\sin\theta = n\lambda$

$\therefore a\sin\theta = n\lambda$ ← 可由 x-ray 繞射得知

(如上圖, 入射為↓)

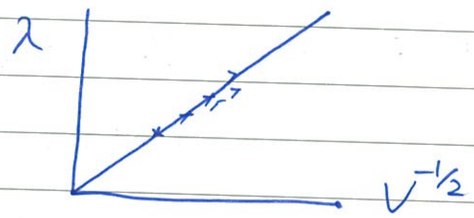
$V = 54V$



expt. $\lambda = 0.215\sin 50^\circ = 1.65\,A^\circ$

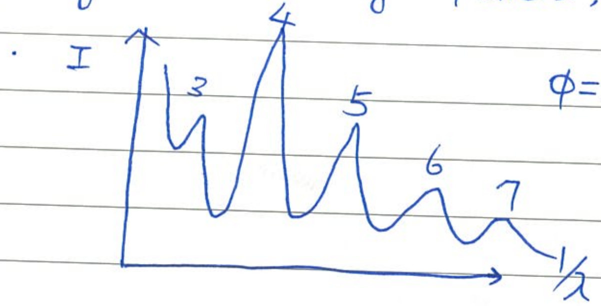
theory. $\lambda = \frac{h}{\sqrt{2mE}} = 1.67\,A^\circ$, E ← 54eV

V changes → 400 V



注意, given any V (thus λ), 不一定有對應之θ! 這是因為不一定能找到 reflecting planes!

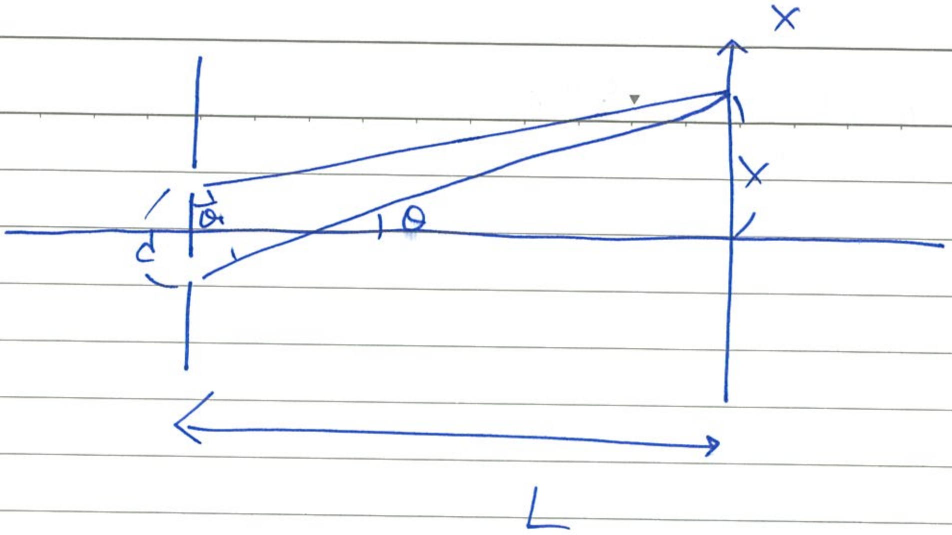
為此, Davisson 和 Germer 固定 θ (φ), 改變 V (λ), 若 θ corresponding to reflecting planes, one sees



$\phi = 50^\circ$ $n = 3, 4, 5, 6, \ldots$

比較直接的干涉實驗為双狹縫干涉實驗, 但由於要準備合入相似之双狹縫不易, 直到最近才成功 (Fig. I)

Paper House



$d\sin\theta = n\lambda$ 亮紋

$\sin\theta \sim \theta \sim \tan\theta \sim \frac{x}{L}$ ∴ $x_n = n\frac{L\lambda}{d}$

$\Delta x_n \simeq \frac{L}{d}\lambda$.

課本上. $\Delta x_n \sim 50\ \mu m$ (neutron)

$\lambda \simeq 18.5\ Å$ > $\frac{L}{d} \sim \frac{\Delta x_n}{\lambda}$ $Å \simeq 10^{-10} m$

if $L \sim 1m$ $\sim \frac{5\times10^5 Å}{2\times10 Å} \sim 10^4$

$d \sim \frac{L}{10^4} \sim 0.1\ mm$

电子也可作同樣的實驗，但 required 比較小！

⇒ Fig. I.

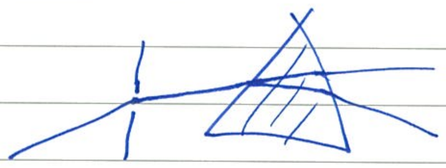
以上建立了粒子之波的特性！

但這些波之精確特性為何？ We'll come back to this point at the end of this chapter.

Atomic spectra (原子光譜)

18世紀末，研究由元素燃燒 (flame) 或在氣體中放電所得之光譜為一熱門的課題。

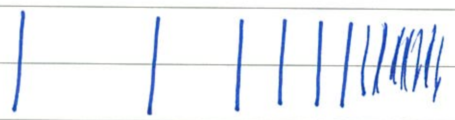
那時已發現，這些光若經過 prism 可被分開，而且



不像白光，為 discrete lines！

主要的 breakthrough 為 1885年，J. Balmer 發現氫的可見光與近紫外光的光譜可以用經驗式 $\lambda_n = 364.6 \frac{n^2}{n^2-4}$ nm 來表示，

(Balmer series) $n=3,4,5,6\ldots$



Later, it was generalized to by Rydberg – Ritz

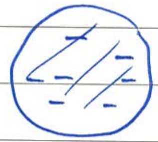
$$\underbrace{\frac{1}{\lambda} = R}_{\text{Rydberg constant}}\left(\frac{1}{m^2}-\frac{1}{n^2}\right) \quad n>m \quad \text{(Rydberg–Ritz formula)}$$

這些 discrete spectrum 是很難以古典電磁學來了解的，為了了解它的起源，必須要有一個原子的模型

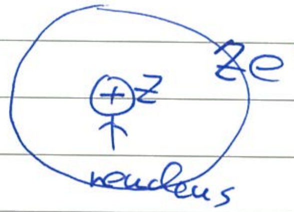
The Bohr Atom

* 氫原子為最簡單的原子。其模型由早期之 J.J. Thomson 的 model

plum pudding



經 Rutherford 以 α-particle (He之原子核) 撞擊實驗後，確定之為



但問題是電子以什麼方式在原子中運動？
這些和原子光譜有何關係？

由於電荷是以 $U = \frac{+q_1 q_2}{r}$ 的庫侖力作用，

正如行星運動一般，naively, one expects that electrons will follow the same orbits.

問題是古典電磁學告訴我們加速的電荷 ⇒ radiate

∴ eventually,  collapses!

How do we compromise this?

在1913，Niels Bohr 提出了一個簡單的模型，capture 主要的 feature，所以，雖然 the model 並非最終的模型，還是值得一提

設定 capture 了重要 feature

$\left\{ \begin{array}{l} r \\ \Delta E \propto \frac{1}{n^2} \end{array} \right.$

Note: 就某種意義來說 Bohr 的模型可說是 Rydberg-Ritz 公式之重建，賦予物理上、運動上之意義

$\boxed{\leftarrow \text{2 weeks}}$

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* Essential points in the Bohr model

① 存在穩定且不 radiation 的軌道 (stationary)

$L = \frac{h}{2\pi} \times n$. (其意義在 De Broglie 提出物質波後更清楚。)

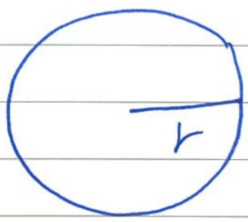
$$mvr = n\frac{h}{2\pi} = \hbar = 1.0545 \times 10^{-27} \text{ erg sec}$$

② transitions from one allowed orbit (E) to another (E')

$$\frac{E-E'}{h} = \nu \leadsto \quad E-E' = h\nu = \hbar\omega$$

$\boxed{\omega = 2\pi\nu}$

or absorb $\frac{E'-E}{h} = \nu$



$$mvr = \frac{nh}{2\pi}$$

以 De Broglie 來了解

$$\boxed{\Rightarrow 2\pi r = n\frac{h}{p} = n\lambda \quad \text{駐波 } (\because \text{stationary!})}$$

由 $\boxed{\left(\frac{1}{4\pi\epsilon_0}\right)\frac{Ze^2}{r^2} = \frac{mv^2}{r}}$. $v = \frac{Ze^2}{mrv} = \frac{Ze^2}{n\,(h/2\pi)} \left(\frac{1}{4\pi\epsilon_0}\right)$ $\hbar$

$$\therefore r = \frac{nh}{2\pi mv} = \frac{1}{4\pi^2}\frac{n^2h^2}{Ze^2m}(4\pi\epsilon_0)$$

$= -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r}$

$$E = \underbrace{\frac{1}{2}mv^2}_{} - \frac{Ze^2}{r} = -\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{2\pi^2 e^4 Z^2 m}{h^2 n^2} = -\frac{1}{2}mc^2\left(\frac{Z\alpha}{n}\right)^2$$

$-\frac{1}{2}mv^2$

$= -\frac{1}{2}m\left(\frac{cZ\alpha}{n}\right)^2$ $\quad L - \frac{Ze^2}{2r}$

$\alpha = e^2/\hbar c\left(\frac{1}{4\pi\epsilon_0}\right)$ = fine structure constant

It's convenient to introduce α : $\frac{v}{c} = \frac{Z\alpha}{n}$

α: dimensionless, $\left[\frac{e^2}{4\pi\epsilon_0}\right] =$ Energy $\cdot$ L

$$[\hbar c] = [\hbar\omega \underbrace{\frac{c}{\omega}}_{\lambda}] = \text{Energy} \cdot L$$

$$\text{Value} = \frac{(1.6\times10^{-19}\ \text{Coulomb})^2}{\underbrace{1.0545\times10^{-34}}_{\hbar} \times 3\times10^8} \times 8.988\times10^9$$

$$\simeq \frac{1}{137} \quad (\text{MKS})$$

Gaussian, $\alpha = \frac{e^2}{\hbar c}$ $\quad e = 1.6\times10^{-19} \times 3\times10^9$ Stat Coulombs $= 4.8\times10^{-10}$ Stat Coulombs

$$= \frac{(4.8\times10^{-10})^2}{\underbrace{1.0545\times10^{-27}}_{\text{erg}\cdot\text{sec}} \cdot 3\times10^{10}\ \text{cm/sec}} \simeq \frac{1}{137}$$

Note that in the atomic physics, the following units are convenient: (but not essential)

① mc^2 (energy) ~ 0.51 MeV

② $\frac{\hbar}{mc}$ (compton length/2π) $\sim 3.8\times10^{-11}$ cm

③ $\frac{\hbar}{mc^2}$ (time) $\sim 1.3\times10^{-21}$ sec

④ mc (momentum)

* More results from the Bohr theory:

① radius

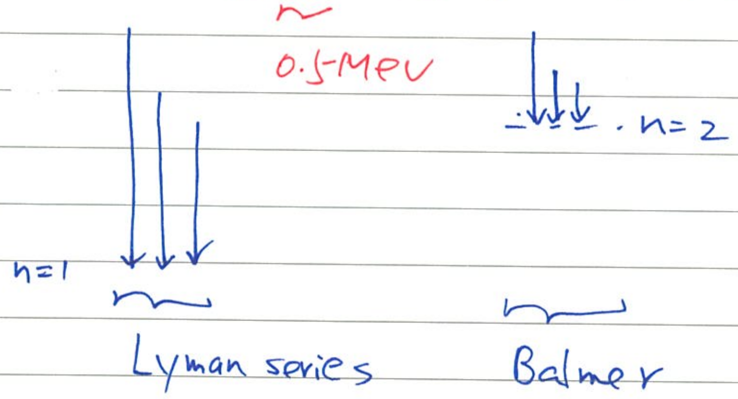
$$r = \frac{1}{4\pi^2}\frac{n^2h^2}{Ze^2m} \underset{n=1}{=} \frac{1}{Z}\boxed{\frac{\hbar}{mc}}\frac{\hbar c}{e^2} = \frac{n^2}{Z\alpha}\lambda_{comp.}$$

$$= \frac{137}{Z}\frac{\hbar}{mc} = \frac{0.53}{Z}\,\text{Å}$$

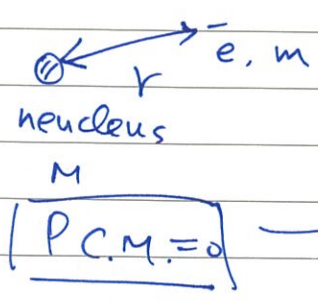
$Z=1$, $n=1$, $r = a_0$ = Bohr radius

② binding energy ⇒ $n=1$ to $n=\infty$

$$\therefore = \frac{1}{2}\underbrace{mc^2}_{0.5\,MeV}(Z\alpha)^2 = 13.6\,Z^2\,eV \qquad \alpha = \frac{1}{137}$$



③ Reduced mass correction



$P = p$

$$E_K = \frac{P^2}{2M} + \frac{p^2}{2m} = \frac{1}{2(M+m)}P_{cm}^2 + \frac{1}{2\mu}p^2$$

$$= \frac{1}{2\mu}p^2$$

$p = \mu\dot{r}$

$$\frac{1}{\mu} = \frac{1}{M} + \frac{1}{m}$$

$$\mu = \frac{m}{1+m/M} \qquad \therefore \text{replace } m \text{ by } \mu!$$

$m/M \sim \frac{1}{2000}\left(\frac{1}{1836}\right)$

example : positronium atom

$+e$ $\qquad$ e^- $\qquad\qquad$ $\mu = \frac{m}{2}$

binding energy $= \frac{1}{2}\mu c^2 \alpha^2 = \frac{13.6}{2}\,eV = 6.8\,eV$

but $r = \frac{n^2 h^2}{4\pi^2 Z e^2 m}$ becomes double
$m \to \mu$

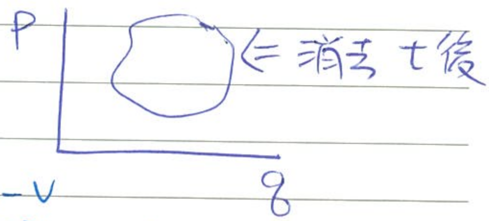
* Bohr's quantization condition was later generalized by Wilson & Sommerfeld to

$$\oint p\,dq = nh \qquad n = \text{integers}.$$

$p = \frac{h}{\lambda}$ $\oint \frac{dq}{\lambda} = n$

for any coordinate q which is periodic in time.

p is the corresponding momentum. $\left(= \frac{\partial \mathcal{L}}{\partial \dot{q}}\right)$



example: $q = \theta$

$E_k - V$

$p = L = I\dot{\theta} \qquad \left(\mathcal{L} = \frac{1}{2}I\dot{\theta}^2 - V\right)$

$\therefore \oint L\,d\theta = 2\pi L = nh \qquad L = \frac{nh}{2\pi}$

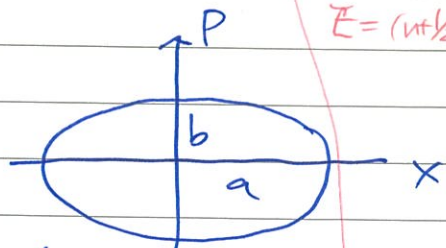
Planck
$E = n\hbar\omega$

More example : SHO

exact
$E = (n+\frac{1}{2})\hbar\omega$

$$E = \frac{p_x^2}{2m} + \frac{1}{2}kx^2$$

$$\underbrace{\frac{p_x^2}{2mE}}_{\substack{\parallel \\ b^2}} + \underbrace{\frac{x^2}{2E/k}}_{\substack{\parallel \\ a^2}} = 1$$



$\oint p\,dx =$ area (HW. Ex. 15)

$= \pi ab = \pi\sqrt{2mE}\sqrt{\frac{2E}{k}} = nh$

以上一般稱為 Old quantum mechanics

* ~~The~~ Correspondence principle

① In the old "quantum theory", a way to check the validity of prediction is to consider a limit in which the prediction should be consistent with the classical result.

The idea is formulated as the correspondence principle. Technically, the limit (the classical limit) is $n \to \infty$ (quantum # $\to \infty$).

example: jump from $n+1 \to n$

$$\frac{E_{n+1}-E_n}{h} = \nu = \frac{1}{2\pi\hbar}\frac{mc^2}{2}(Z\alpha)^2\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]$$

$n\to\infty$ 時 $\frac{1}{n^2}-\frac{1}{(n+1)^2} = \frac{1+2n}{n^2(n+1)^2} \to \frac{2}{n^3}$

$$\therefore \frac{E_{n+1}-E_n}{h} = \nu \to \frac{1}{2\pi\hbar}\, mc^2\, \frac{(Z\alpha)^2}{n^3}$$

一致!

classically, $\nu_{cl} = \frac{v}{2\pi r} = \frac{Z\alpha c}{n}\cdot\frac{2\pi Z e^2 m}{h^2\hbar^2}$

$$= \frac{Z\alpha c}{n}\cdot\frac{Ze^2 m}{2\pi n^2\hbar^2}$$

$r = \frac{n^2}{Z\alpha}\lambda_{comp}$

$\nu_{cl} = \frac{1}{2\pi}\frac{Z\alpha c}{n}\frac{Z\alpha}{n^2}\frac{1}{\lambda_{comp}} = \frac{Z\alpha c}{n}\,\frac{Zmc}{n^2 2\pi\hbar}\left(\frac{e^2}{\hbar c}\right) = \frac{(Z\alpha)^2 mc^2}{2\pi\hbar\, n^3}$

$\frac{e^2}{\hbar c} = \alpha$

$= \frac{(Z\alpha)^2}{n^3}\, c\, \frac{mc}{2\pi\hbar}$

② <u>Comment on selection rules</u>:

在 quantum 中有所謂的 selection rules:

實驗上，並非所有 transitions ($n \to m$) 都被允許，被允許之規則叫 selection rules.

Correspondence principle 之另一部分 假設

selection rules can be applied in the entire range of quantum numbers.

<u>example</u>: 在之前之 example 中，

$$\frac{E_{n+2}-E_n}{h} \xrightarrow[n\to\infty]{} 2\nu_{cl} \quad 不 \neq \nu_{cl}$$

$\therefore$ $n+2 \to n$ is forbidden in $n \to \infty$

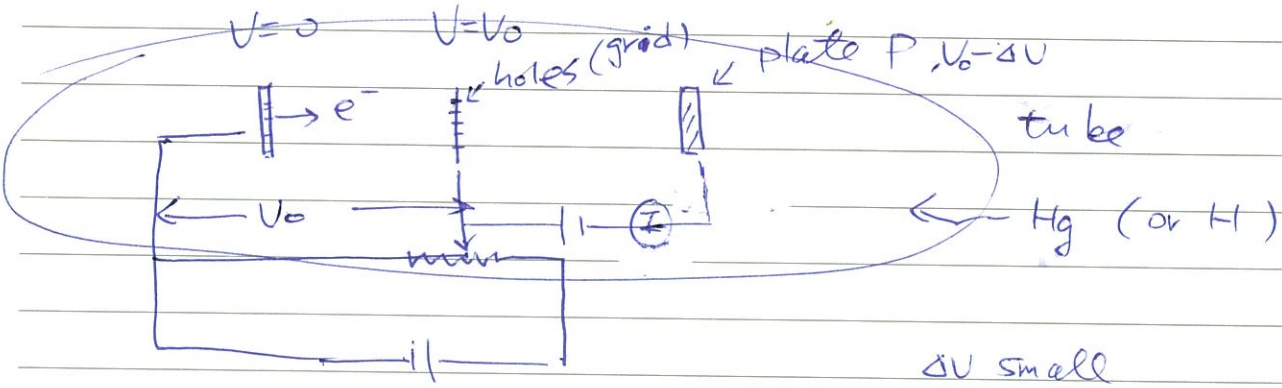
One infers from this: selection rules $n \to n \pm 1$!

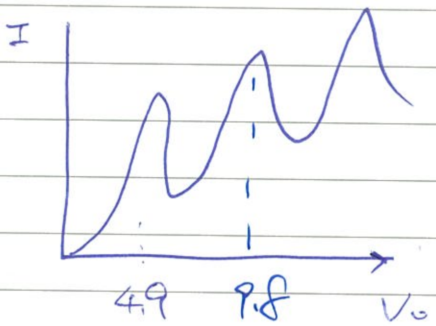
這個 selection rule 在 Chapter 21 會驗証

(只對 circular orbits 才成立)

* The Franck - Hertz expt.

The energy quantization of electrons in atoms was directly probed non-optically ! by J. Franck & G. Hertz





⇈ first excited state of Hg, ⇈ multiple excitation of first excited state

e^- inelastic scattered off Hg !

Check : Hg (excited) can go back to the ground ⇝ $\lambda = \frac{c}{\nu} = \frac{hc}{eV_0} = 253 nm$

indeed, one can see such a line in the spectrum when $V_0 > 4.9 V$!

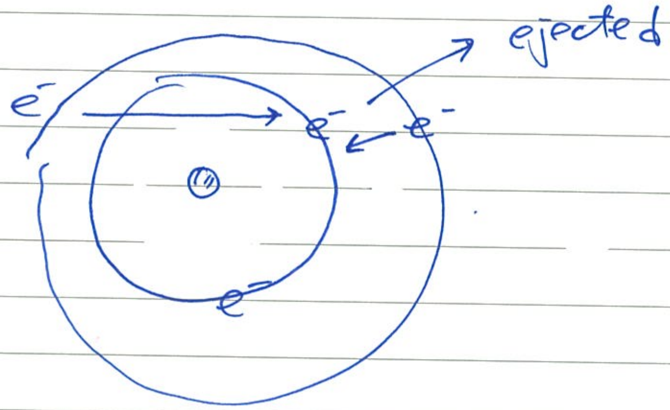
2nd excited state (6.7 eV) is not observed due to low cross section in comparison to the 1st one

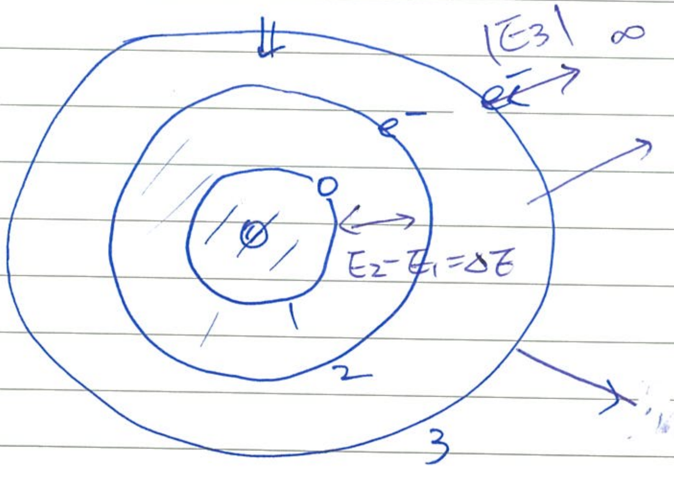
(before e^- can reach 6.7 eV, it excites 4.9 already)

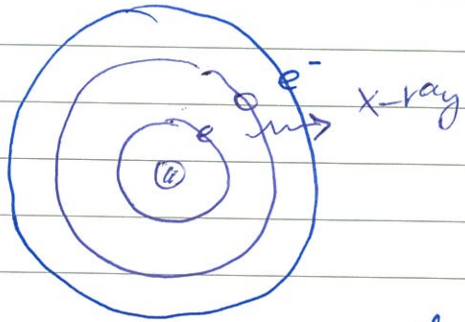
Origin of
* X-ray sharp line spectra

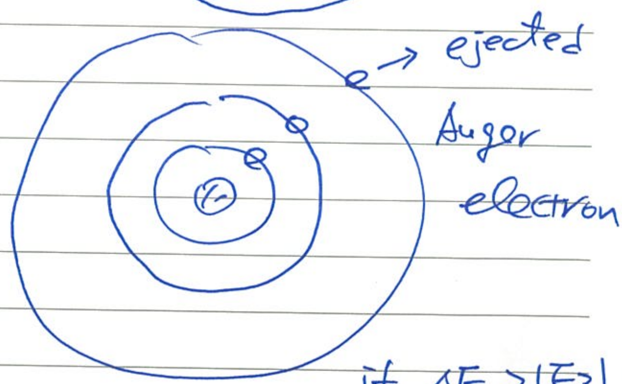
The origin of sharp X-ray lines is also due to the discrete levels of electrons in the atoms.

This is essentially what happens:









if $\Delta E > |E_3|$

$\leftarrow$ 10 hours

Particle - wave duality & the compromise

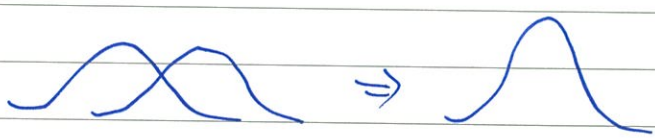
Remarkable facts about the two-slit expts.

① $I \propto$ # of photons

② Reducing intensity of incoming light but accumulating spots of light on the screen
$\Rightarrow$ same pattern of I.

③ if one tries to pin down which slit the photon goes through

$\Rightarrow$ classical result



Intensity $\propto$ # of photons seem to be an important hint to bridge wave & particle nature:

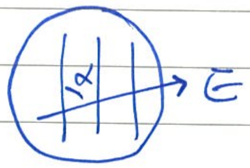
first, what does the intensity imply for individual photon?

$\because$ Each photon in the two-slit expt. behaves randomly, and only after the accumulation of many photons $\Rightarrow$ we have a pattern of I

∴ 所以 assign $I \propto$ probality for each photon to arrive at one particular point ⇓ $\boxed{I: \text{guide photon!}}$

這樣似乎可以合光的一些行為一致:

如 ① polarized light

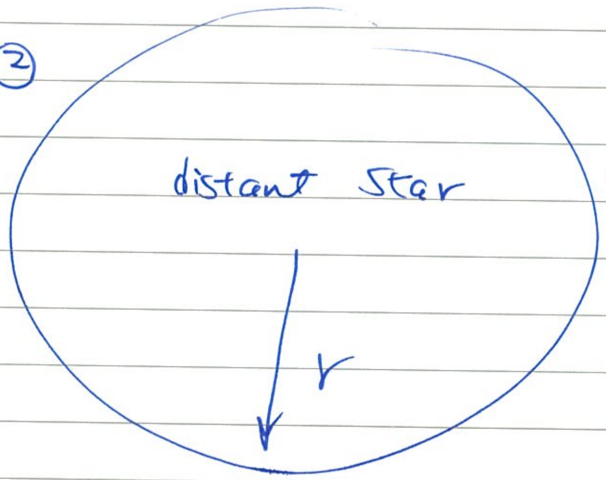


通過之 $I = I_0 \cos^2\alpha$

以單一 photon 而言 $\left\{ \begin{array}{l} \text{either blocked} \\ \text{or go through} \end{array} \right.$

其機會 $\propto \cos^2\alpha$

②



∵ $I \propto \frac{1}{r^2}$

所以光會被稀釋

wave: 光能轉散布在 $4\pi r^2$ 上

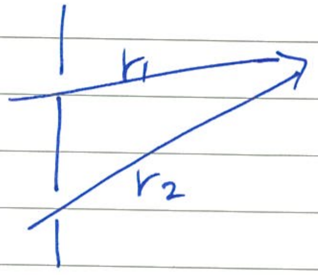
but how can we dectect it by seeing the collapse point on our retina?

以上告訴我們 I 為找到 photon 之機率!

同樣的道理, 對於電子來說 I 也為找到粒子之機率

∴ 問題是 intensity 中之波動行為源自何處？

对於光来說，$I \propto |\vec{E}|^2 \propto |\vec{E}_1+\vec{E}_2|^2$ 电磁波 wave



由 $\vec{E}_1$ 和 $\vec{E}_2$ 之干涉

產生波動的行為

$$|e^{ikr_1}+e^{ikr_2}|^2$$

$$= |e^{ikr_1}|^2 + |e^{ikr_2}|^2 + e^{i(kr_1-kr_2)} + e^{-i(kr_1-kr_2)}$$

对於电子呢？

<u>The probability interpretation</u> (Chapt. 3. p. 42 - 43)

* probability & probability density

discrete variables ⇒ probability

如骰子的点数 $P_i = \frac{1}{6}$

Continuous variables ⇒ probability density (measure)

如 X=random variable $0 \le X \le 1$

$P(x)dx$ = 出現在 $(x, x+dx)$ 之机率

probability density

* Max Born's probability interpretation

$\underbrace{P(x,t)}_{\text{probability density}}\,dx$ = $\underline{\text{probability of finding the particle in } (x, x+dx)}$

$= |\Psi(x,t)|^2 dx$ i.e. $P(x,t) = |\Psi(x,t)|^2$

$\Uparrow$

物質波 $\Leftrightarrow$ $\vec{E}(x,t)$

Complex

目前先假設 多有
1個常數

$\because \int_{-\infty}^{\infty} P(x,t)\,dx = 1$ $\therefore \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$

(square integrable)