Principle and application of Quantum Technology
Homework 4 Due: June 5, 2025

Ex 1 Consider a coupled system of a transmon and an LC resonator as shown in Fig.1. Asnwer the following questions.

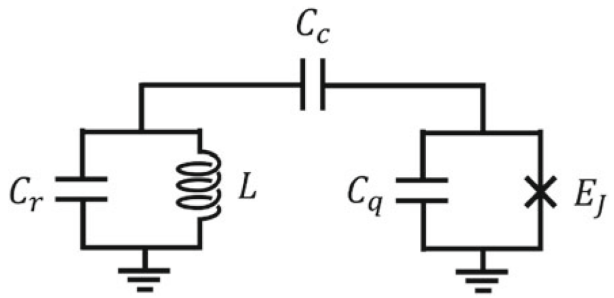

FIG. 1: Coupled system of a transmon and an LC resonantor

(a) 5% Suppose θ describes the phase of the Josephson junction. By using θ and the Josephson energy E_J, the current operator I_r passsing through L, charge operators Q_r and Q_q on C_r amd C_q respectively, find the Hamiltonian that characterizing the whole system.

(b) 10% By expanding $\cos\theta = 1 - \theta^2/2! + \theta^4/4!$ to $O(\theta^4)$, H can be expressed in terms of the creation and annhiliation operators $c^\dagger$, c, $a^\dagger$, and a as

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_q a^\dagger a + \frac{\hbar\alpha}{2} a^\dagger a^\dagger a a - \hbar g(c^\dagger + c)(a^\dagger + a),$$

find ω_c, ω_q, α and g.

Ex. 2 10%

Apply the quantum Fourier transform U_{QFT} to the state $|\psi\rangle = \frac{1}{2}\sum_{j=0}^{7}\cos(2\pi j/8)|j\rangle$. That is, find $U_{QFT}|\psi\rangle$. Here $|j\rangle$ $(j = 0, 1, 2, ..., 7)$ are the basis states in 3 qubit Hilbert space with j represeting the binary number in decimal representation. For instance, $|7\rangle = |1\rangle\otimes|1\rangle\otimes|1\rangle$. $U_{QFT} = \frac{1}{2\sqrt{2}}\sum_{j=0}^{7}\sum_{k=0}^{7} e^{-i2\pi kj/8}|k\rangle\langle j|$.

Ex.3

Let $B_1 = \{|H\rangle, |V\rangle\}$ denote an orthonormal basis in the Hilbert space of a photon. Here the state $|H\rangle$ and $|V\rangle$ represent the state of a photon with the horizontal and vertical polarization. Let $B_2 = \{|\phi_0\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), |\phi_1\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)\}$ denote a second orthonormal basis in the Hilbert space of a photon. These states are identified with the 45° and -45° polarization of a photon. Alice sends photons randomly prepared in one of the four states $|H\rangle$, $|V\rangle$, $|\phi_0\rangle$ and $|\phi_1\rangle$ to Bob. Bob then randomly chooses a basis B_1 or B_2 to measure the polarization of the photon. All random decisions follow the uniform distribution. Alice and Bob interpret $|H\rangle$ as binary 0 and $|V\rangle$ as binary 1 in the basis B_1. They interpret $\phi_0\rangle$ as binary 0 and $|\phi_1\rangle$ as binary 1 in the basis B_2.

(a) 10% What is the probability that Bob measures the photon in the state prepared by Alice, i.e. what is the probability that the binary interpretation is identical for Alice and Bob?

(b) 10% An eavesdropper (named Eve) intercepts the photons sent to Bob and then resends a photon to Bob. Eve also detects the photon polarization in one of the bases B1 or B2 before resending. What is the probability that the binary interpretation is identical for Alice and Bob?

Ex 4 10% page 233, problem 10-10

Ex.5 5% Page 233, problem 10-4