

Principle and application of Quantum Technology

Homework 1 Due: March 20, 2025

Ex.1 Delayed Quantum Eraser

Consider the double-slit experiment as set up shown in Fig. 1, in which a 1-bit (with two state $|\uparrow\rangle$ and $|\downarrow\rangle$) quantum path detector sitting in the path of the particle passing through a double-slit. The 1-bit quantum path detector serves as the which-way detector: when the 1-bit detector is in the state $|\uparrow\rangle$, the particle goes through A -slit and is the state $|\psi_1\rangle$; otherwise, the 1-bit detector is in the state $|\downarrow\rangle$, the particle goes through B -slit and is the state $|\psi_2\rangle$.

(a) 5% Let the combined state of the particle and the which-way detector be $|\psi\rangle$. Find normalized state $|\psi\rangle$ in terms of $|\psi_i\rangle$, $|\uparrow\rangle$, and $|\downarrow\rangle$.

(b) 10% Find the probability density for detecting the particle at the position x on the screen. Show that it does not exhibit interference. Hence the which-way detector destroys the interference.

(c) 10% A quantum eraser is introduced by rotating the 1-bit detector to detect $|\pm\rangle$ state with $|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$. Show that interference will show up in the probability density $|\langle +|\psi(x)\rangle|^2$ or $|\langle -|\psi(x)\rangle|^2$.

For the case of delayed quantum eraser in this setup, it corresponds to the situation that one looks at the which-way detector after the particle hits the screen. Would the probability density $|\langle +|\psi(x)\rangle|^2$ or $|\langle -|\psi(x)\rangle|^2$ change? This explains the results of delayed quantum eraser that we showed in class.

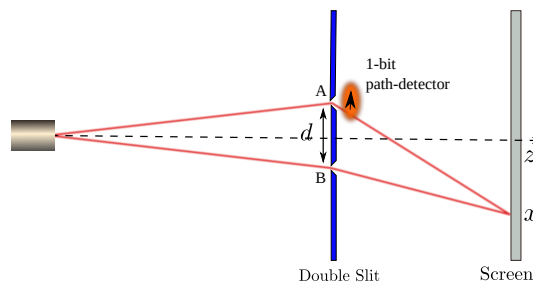


FIG. 1: Schematic plot of a two-slit interference experiment in the presence of a 1-bit which-way detector.

Ex.2 When an operator \hat{A} satisfies $\hat{A}^2 = \hat{I}$,

(a) 10% show that $\hat{U} \equiv e^{i\theta\hat{A}} = \cos\theta\hat{I} + i\sin\theta\hat{A}$ by using the definition of the exponential

of a matrix, $e^{\hat{A}} \equiv \sum_{n=0}^{n=\infty} \frac{\hat{A}^n}{n!} = \hat{I} + \hat{A} + \frac{\hat{A}^2}{2!} + \frac{\hat{A}^3}{3!} + \dots$.

(b) **10%** If \hat{A} is Hermitian, find \hat{U}^\dagger and \hat{U}^{-1} . Is \hat{U} unitary?

Ex.3 20% page 71 problem 3-8.

Ex.4 10% Page 72, problem 3-9

Ex.5 The state space of a certain physical system is three-dimensional. Let $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$ be an orthonormal basis of this space. The kets $|\psi_1\rangle$ and $|\psi_2\rangle$ are defined by :

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}|u_1\rangle + \frac{i}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle \\ |\psi_2\rangle &= \frac{1}{\sqrt{3}}|u_1\rangle + \frac{1}{\sqrt{3}}|u_2\rangle \end{aligned}$$

(a) **5%** Are these kets normalized?

(b) **5%** What is $\langle\psi_1|\psi_2\rangle$?

Ex.6 10% Suppose that we do a measurement of the observable \hat{O} on some particle and get the value α . Using the concept of "collapse of state", argue that after the measurement, the state of the particle has to be an eigenstate of \hat{O} with eigenvalue of α .