# Principle and application of Quantum Technology
# Homework 1 Due: March 22, 2024

## Ex.1 Delayed Quantum Eraser

Consider the double-slit experiment as set up shown in Fig. 1, in which a 1-bit (with two state $|\uparrow\rangle$ and $|\downarrow\rangle$) quantum path detector sitting in the path of the particle passing through a double-slit. The 1-bit quantum path detector serves as the which-way detector: when the 1-bit detector is in the state $|\uparrow\rangle$ , the particle goes through $A$-slit and is the state $|\psi_1\rangle$; otherwise, the 1-bit detector is in the state $|\downarrow\rangle$ , the particle goes through $B$-slit and is the state $|\psi_2\rangle$.

**(a) 5%** Let the combined state of the particle and the which-way detector be $|\psi\rangle$. Find normalized state $|\psi\rangle$ in terms of $|\psi_i\rangle$, $|\uparrow\rangle$, and $|\downarrow\rangle$.

**(b) 10%** Find the probabilty density for detecting the particle at the position $x$ on the screen. Show that it does not exhibit inteference. Hence the which-way detector destroys the interference.

**(c) 10%** A quantum eraser is introduced by rotating the 1-bit detector to detect $|\pm\rangle$ state with $|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$. Show that inteference will show up in the probaility density $|\langle +|\psi(x)\rangle|^2$ or $|\langle -|\psi(x)\rangle|^2$.

For the case of delayed quantum eraser in this setup, it corresponds to the situaton that one looks at the which-way detector after the particle hits the screen. Would the probaility density $|\langle +|\psi(x)\rangle|^2$ or $|\langle -|\psi(x)\rangle|^2$ change? This explains the results of delayed quantum eraser that we showed in class.

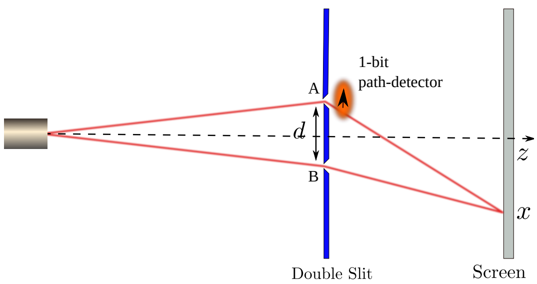


FIG. 1: Schematic plot of a two-slit interference experiment in the presence of a 1-bit which-way detector.

**Ex.2** When an operator $\hat{A}$ satisfies $\hat{A}^2 = \hat{I}$,

**(a) 10%** show that $\hat{U} \equiv e^{i\theta\hat{A}} = \cos\theta\hat{I} + i\sin\theta\hat{A}$ by using the definition of the exponential of a matrix, $e^{\hat{A}} \equiv \sum_{n=0}^{n=\infty} \frac{\hat{A}^n}{n!} = \hat{I} + \hat{A} + \frac{\hat{A}^2}{2!} + \frac{\hat{A}^3}{3!} + \cdots$.

**(b) 10%** If $\hat{A}$ is Hermitian, find $\hat{U}^\dagger$ and $\hat{U}^{-1}$. Is $\hat{U}$ unitary?

**Ex.3 20%** page 71 problem 3-8.

**Ex.4 10%** Page 72, problem 3-9

**Ex.5** The state space of a certain physical system is three-dimensional. Let $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$ be an orthonormal basis of this space. The kets $|\psi_1\rangle$ and $|\psi_2\rangle$ are defined by :

$$
\begin{aligned}
|\psi_1\rangle &= \frac{1}{\sqrt{2}}|u_1\rangle + \frac{i}{2}|u_2\rangle + \frac{1}{2}|u_3\rangle \\
|\psi_2\rangle &= \frac{1}{\sqrt{3}}|u_1\rangle + \frac{1}{\sqrt{3}}|u_2\rangle
\end{aligned}
$$

(a) **5%** Are these kets normalized?

(b) **5%** What is $\langle\psi_1|\psi_2\rangle$?

**Ex.6 10%** Suppose that we do a measurement of the observable Ô on some particle and get the value $\alpha$. Using the concept of "collapse of state", argue that after the measurement, the state of the particle has to be an eigenstate of Ô with eigenvalue of $\alpha$.