Homework 1
Due: Oct 19, 2023

Ex.1 10% Consider a ring coil with rectangular cross section. The outer and inner diameters of the ring are 11 and 6 mm, respectively, with the thickness of the ring being 2 mm. The measured current I in the ring is found to decay with the change $\Delta I/I = 0.022$ in 278 hours. Estimate the upper bound of the resistivity for the ring.

Ex.2 10% Consider a supercondiuctor with the shape of hollow cylinder as shown in Fig. 1. Explain how the magnetic flux and supercurrent are distributed in the cylinder during the following experimental procedure:
(1)Field cooled: Applying a magnetic field at temperature $T > T_c$, then cooling the sytem below T_c, after cooling, one then removes the applied magnetic field.
(2) Zero-field cooled: Do not apply magenteic field at temperature $T > T_c$, and directly cool down the superconductor below T_c, then apply a magnetic field.

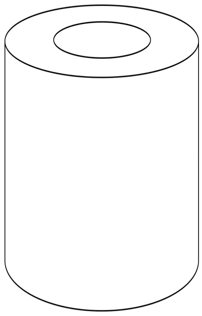

FIG. 1: Superconductor with hollow-cylinder shape

Ex.3 10% The critical current of a superconducting wire
A current I is injected into a long superconduting wire with radius R. Let λ be the penetration length.
In the London model, find the current density $J(r)$ for $r \leq R$. (Express your answers in terms of r, R, λ, and modified Bessel functions). Find the critical current I_c when the magentic field at $r = R$ just becomes the critcal field H_c.

Ex 4 5% By using the thermodynamic relation $dF = -sdT + \frac{1}{4\pi}\int H\delta B d^3\vec{r}$, derive that the condensation energy of a supercondutor is given by $F_S - F_N = \frac{H_c^2}{8\pi}$.

Ex 5 10% By applying the London model to describe the electromagentic dynamics of superconductors, find the peneration depth when the mage=netic field is AC with angular frequency being ω.

Ex 6
(a) 10% Consider a perfect conductor with the transition temperature being T_c. The perfect conductor is cooled across T_c under a uniform magnetic field $B_0\hat{z}$. In $T < T_c$, the magneti field is turned off. Assuming that the perfect conductor occupies the region $x > 0$ with one of it surface being the yz plane, by using the London model to describe the electromagentic dynamics of the perfect conductor, find the current density $\vec{J}(x)$ inside the perfect conductor.
(b) 10% Consider a perfect conductor with the transition temperature being T_c. The shape of the perfect conductor is cylindrical and it is cooled under zero magneytic field across T_c. In $T < T_c$, the perfect conductor is placed above a magnet so that the magneti field is turned on. Assuming that the perfect conductor occupies the region $x > 0$ with one of it surface being the yz plane, by using the London model to describe the electromagentic dynamics of the perfect conductor, find the magentic field $\vec{B}(x)$ inside the perfect conductor. Will the perfect conductor float above the magnet?