Condensed Matter Physics (I): Homework 4
Due: November 27, 2025

Exercises in Ashcroft/Mermin

1-4,1-5,2-1, 30-7

Ex.1 25% The Landau Free energy is constructed as a polynomial of the order parameter $\vec{M}$, consistent with a given set of symmetry requirements. For instance, as shown in the class, if the system is isotropic, the free energy density can be expressed

$$f = A_0|\nabla\vec{M}|^2 + A_2|\vec{M}|^2 + A_4|\vec{M}|^4 + O(|\vec{M}|^6). \tag{1}$$

Construct f up to terms of fourth order in $\vec{M}$ and second order in ∇ with the following symmetries:

(a)Cubic symmetry: invariance under sign reversal and under interchange of any of the order parameter components. Assume that ∇ is independently spherically symmetric.

(b)A system with two n-component order parameters $\vec{M}_1$, $\vec{M}_2$, with independent spherical symmetry for each one and for ∇.

(c) Same as (b), but now $\vec{M}_1$, $\vec{M}_2$, rotate together.

(d) Same as (b), but now assume $n = d$, and ∇, $\vec{M}_1$, $\vec{M}_2$, all rotate together.

(e) 3-state Potts model: $\vec{M} = (M_x, M_y)$ has two components and f is invariant under 120° rotations. Assume that ∇ is independently spherically symmetric.

Ex.2 10% Consider n interstitial atoms in equilibirum with n vacancies in a crystal with N lattice points and N' possible interstitial positions. Show that the energy E to remove an atom from a lattice site to an interstitial position is given by

$$E = k_B T \ln\left[\frac{(N-n)(N'-n)}{n^2}\right]. \tag{2}$$

In the limit of $n \ll N, N'$, find n in terms of E.

Ex.3 20% Consider a system of two types of charge carriers in the Drude model. The two carriers have the same density n and opposite charges (e and $-e$), and their masses and relaxation rates are m_1, m_2 and τ_1 and τ_2, respectively.

(a) Find the magnetoresistance, $\Delta\rho = \rho(H) - \rho(0)$, where H is the magnetic field.

(b) Calculate the Hall coefficient.

Ex.4 Consider free electrons of mass m in a thin film. The width of the thin film is d.

(a) 10% Find the density of states $D(\epsilon)$ and sketch $D(\epsilon)$ versus ϵ.

(b) 10% Find the specific heat due to electrons at low temperatures in the limit $d \to 0$.

(c) 5% Sketch the specific heat due to electrons at low temperatures for finite d.

Ex.5 10% Consider free electrons of mass m in a wire. The cross section of the wire is $d \times d$. Sketch the density of states $D(\epsilon)$ versus ϵ.