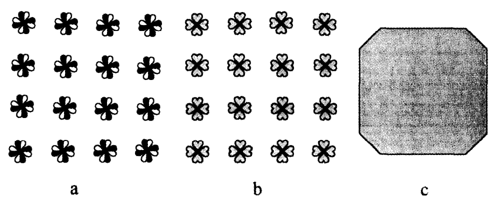
Condensed Matter Physics (I): Homework 1
Due: October 14, 2019

Exercises in Ashcroft/Mermin

4-4,4-7, 6-1

Ex.2 10% Find the reciprocal lattices of bcc and fcc.

Ex.3 10% Fig. 1 shows two "crystals" (a and b) and a polygon (c). Assume that lattices are of infinite sizes, find the point group symmetry operations of three objects. Indicate which lattice has the same point group symmetry as the polygon.



Ex.4 10% Consider a honeycomb lattice with the edge of hexagon be a. If one puts one electron at each lattice point, find the Fourier series expansion of the electron density $\rho(r)$.

Ex.5 10% Consider a collection of particles in three dimensions whose energy is $E = \frac{1}{2}\sum_{i\neq j}\phi(r_{ij})$ with $\phi(r) = \phi_0(\frac{1}{r^3} - 1)e^{-r}$ for $r < 1.5$ and $\phi(r) = 0$ otherwise. Here r_{ij} is the distance in Anstrom between particle i and j. Consider bcc, fcc, and hcp lattices. Which lattice has the lowest overall energy? Find the lowest overall energy.

Ex.6 10% Experiments show that the (200) diffraction peak of C_{60} fcc solid (lattice spacing $a = 14.11A$) is very weak. Assume that the charge distribution of C_{60} can be represented by a surface charge on the surface of sphere of radius $3.5A$. Calculate the form factor of C_{60} molecule in this approximation and show that the form factor of (200) is indeed much less than that of (111).

Ex.7 10% Consider N free and spinless fermions on a line of length L with periodic boundary conditions. Let N, $L \to \infty$ but $n = N/L$ is fixed. Find and sketch the two particle correlation function $C(x)$. Here the two particle correlation function $C(x_1 - x_2)$ is defined as the probability of finding a particle at x_2 when a particle is given at x_1, i.e.,

$$C(x_1 - x_2) = \frac{\text{Probability of finding particles at} x_1 \text{ and} x_2}{\text{Probability of finding a particle at} x_1}. \tag{1}$$