Testing Spin-Statistics Connection by Highly Sensitive Spectroscopy of CO_2

Y.-H. Lien（連育宏）, Y.-L. Hsu（徐毓倫）, Y.-R. Lin（林雁容）, G.-Y. Wu（吳國鈺）, C.-C. Liao（廖俊杰）, J.-T. Shy（施宙聰）

Dept. of Physics, Natl. Tsing Hua Univ., Taiwan

Introduction

Bosons

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Fermions

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Spin-Statistics Connection:

$$\begin{aligned} \text{Boson} \quad &\longleftrightarrow \quad n\hbar \\ \text{Fermion} \quad &\longleftrightarrow \quad (n+1/2)\hbar \end{aligned}$$

Why so Important?

- W. Pauli's 1940 paper proved:
  - By positive energy assumption, half integer spin particles could not be bosons
  - By microcausality (commuting fields for spatially separated points), integer spin particles could not be fermions
  - By M. Fierz 1939's paper, integer spin particles could be bosons and half spin particles could be fermions
  - Pauli reached his famous spin-statistics connection

Why so Important?

Feynman quote
(Feynman Lectures Vol. III, Chapter 4)

An explanation has been worked out by (Wolfgang) Pauli from complicated arguments of Quantum Field Theory and relativity...but we haven't found a way of reproducing his arguments on an elementary level...this probably means that we do not have a complete understanding of the fundamental principle involved...

Question?

How about composite particles?

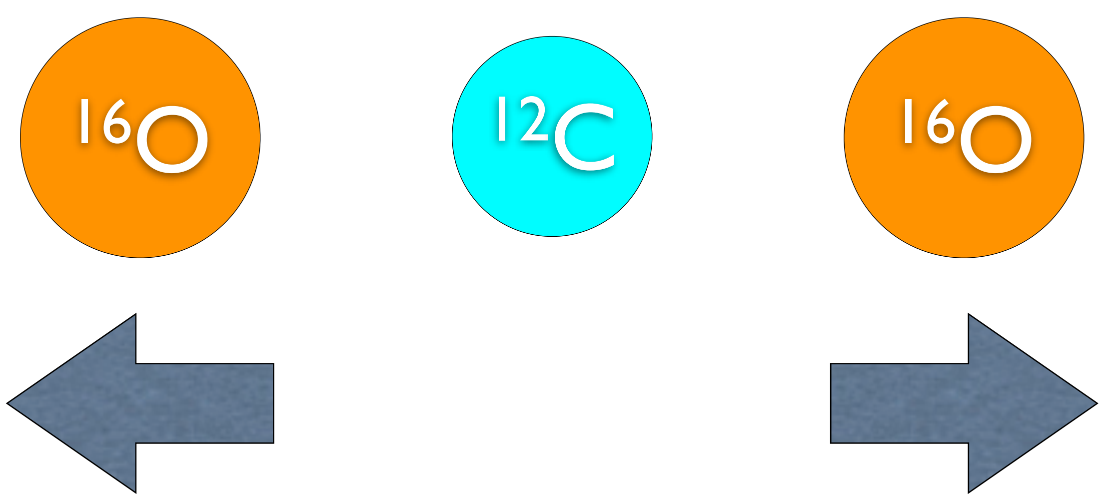
Ehrenfest & Oppenheimer, Physical Review (1931)

From Pauli's exclusion principle we derive the rule for the symmetry of the wave functions in the coordinates of the center of gravity of two similar stable clusters of electrons and protons, and justify the assumption that the clusters satisfy the Einstein-Bose or Fermi-Dirac statistics according to whether the number of particles in each cluster is even or odd. The rule is shown to become invalid only when the interaction between the clusters is large enough to disturb their internal motion.

Forbidden internal states of molecules

Regarding $00^00 \Rightarrow 00^01$ rovibrational transitions of CO_2

The allowed transitions are R(2J) $\quad R \Rightarrow J_{upper} - J_{lower} = +1$



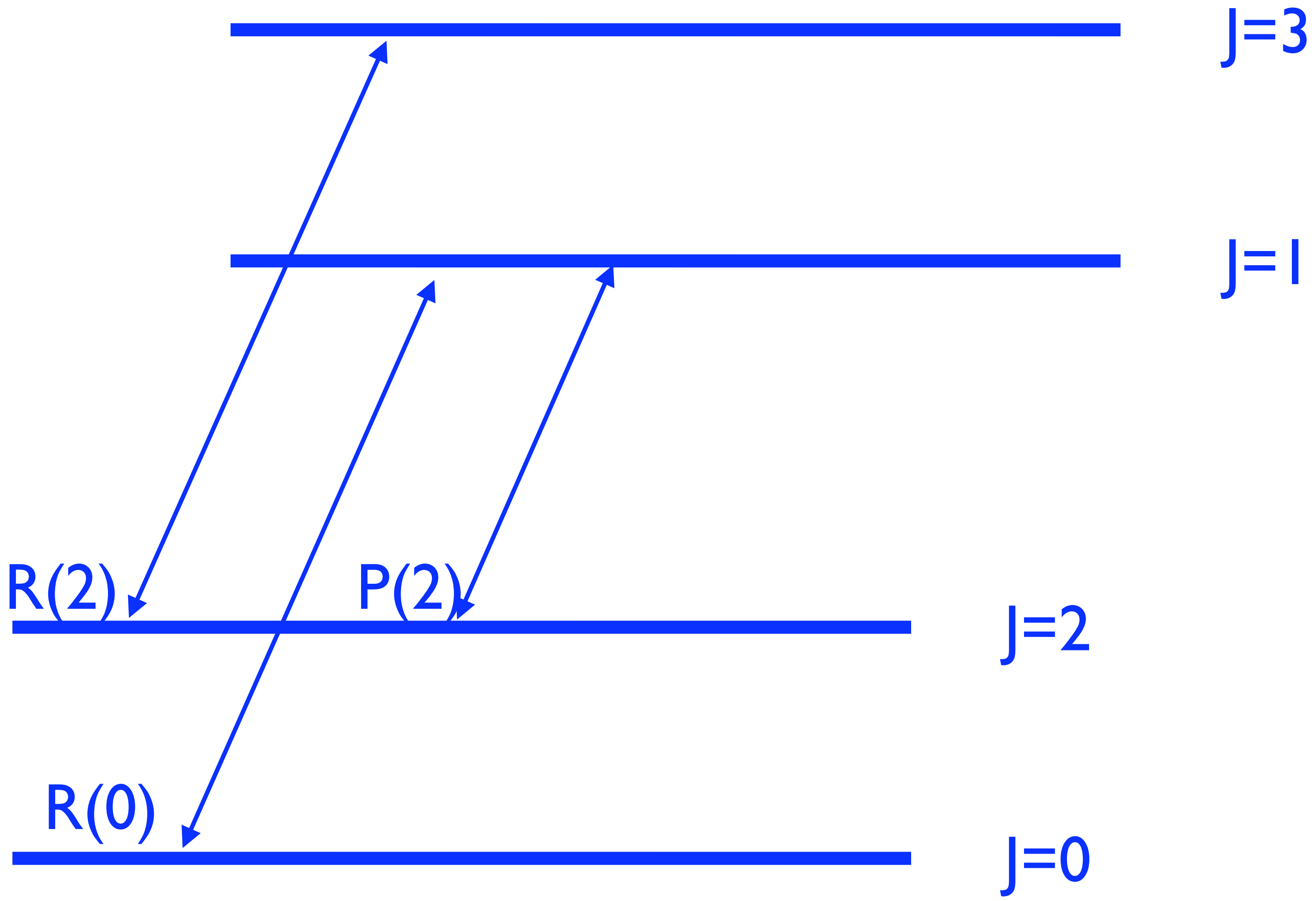
^{16}O nucleus$\Rightarrow$boson

^{16}O wavefuntion$\Rightarrow$symmetric

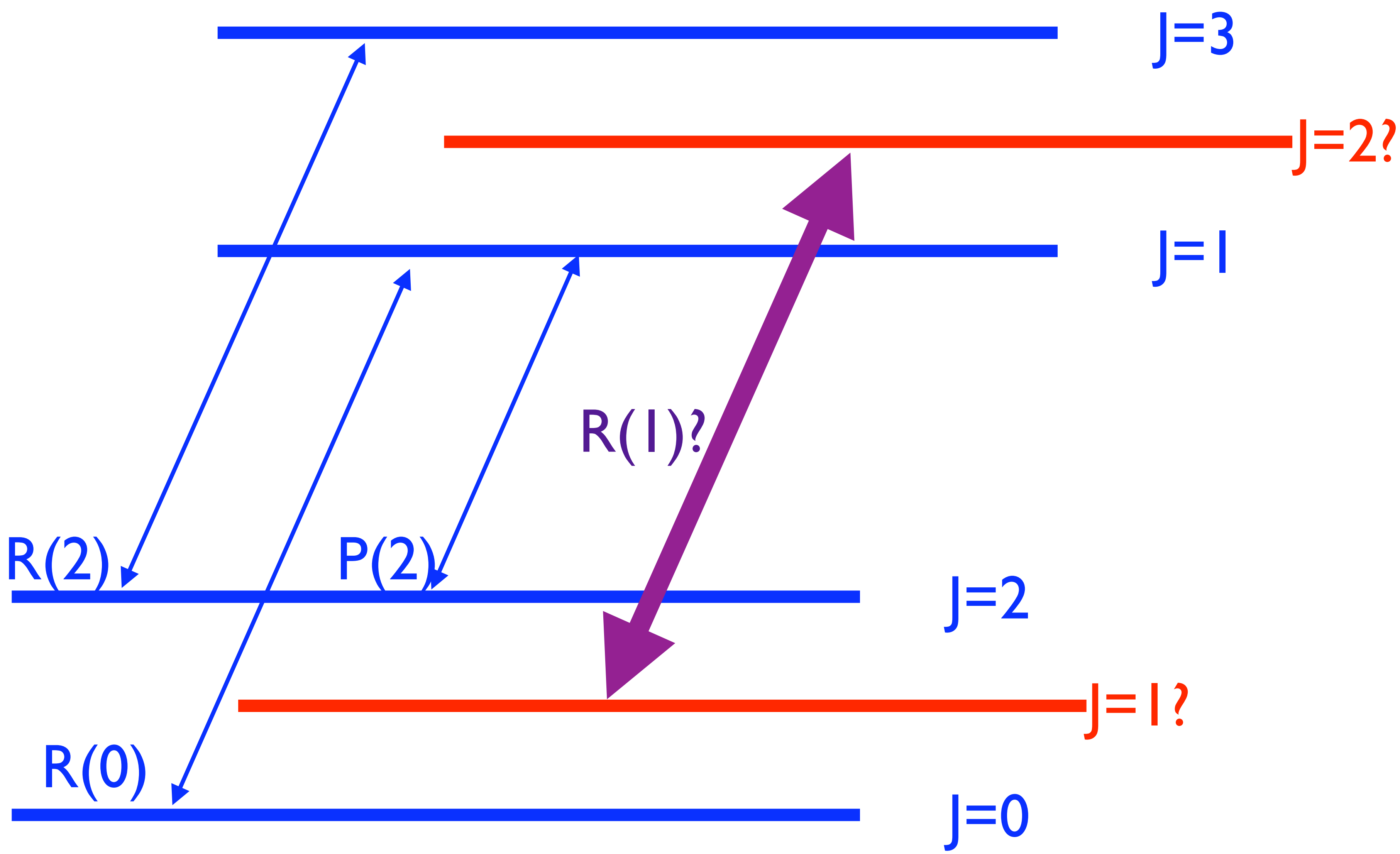
00^00 is symmetric
$\Rightarrow$allowed Js are even

00^01 is anti-symmetric
$\Rightarrow$allowed Js are odd

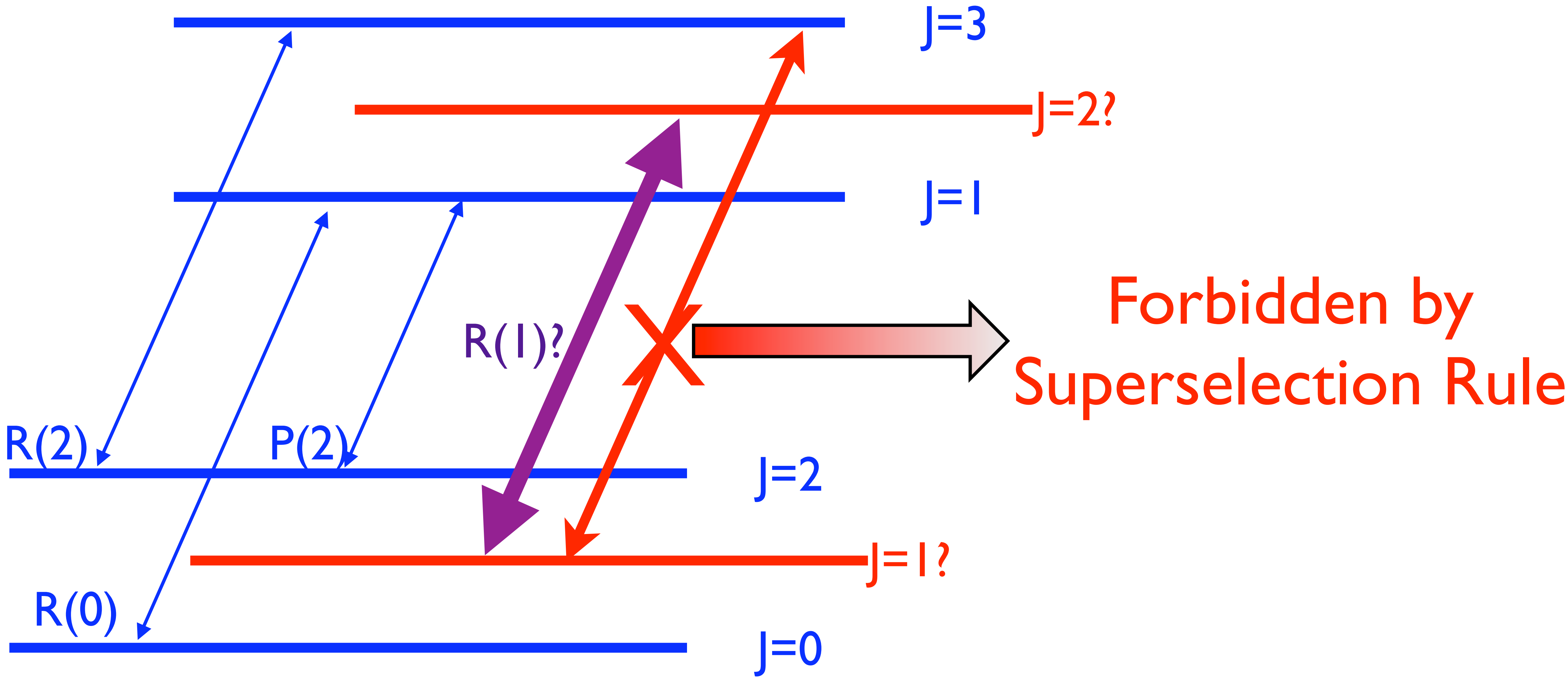
Forbidden internal states of molecules



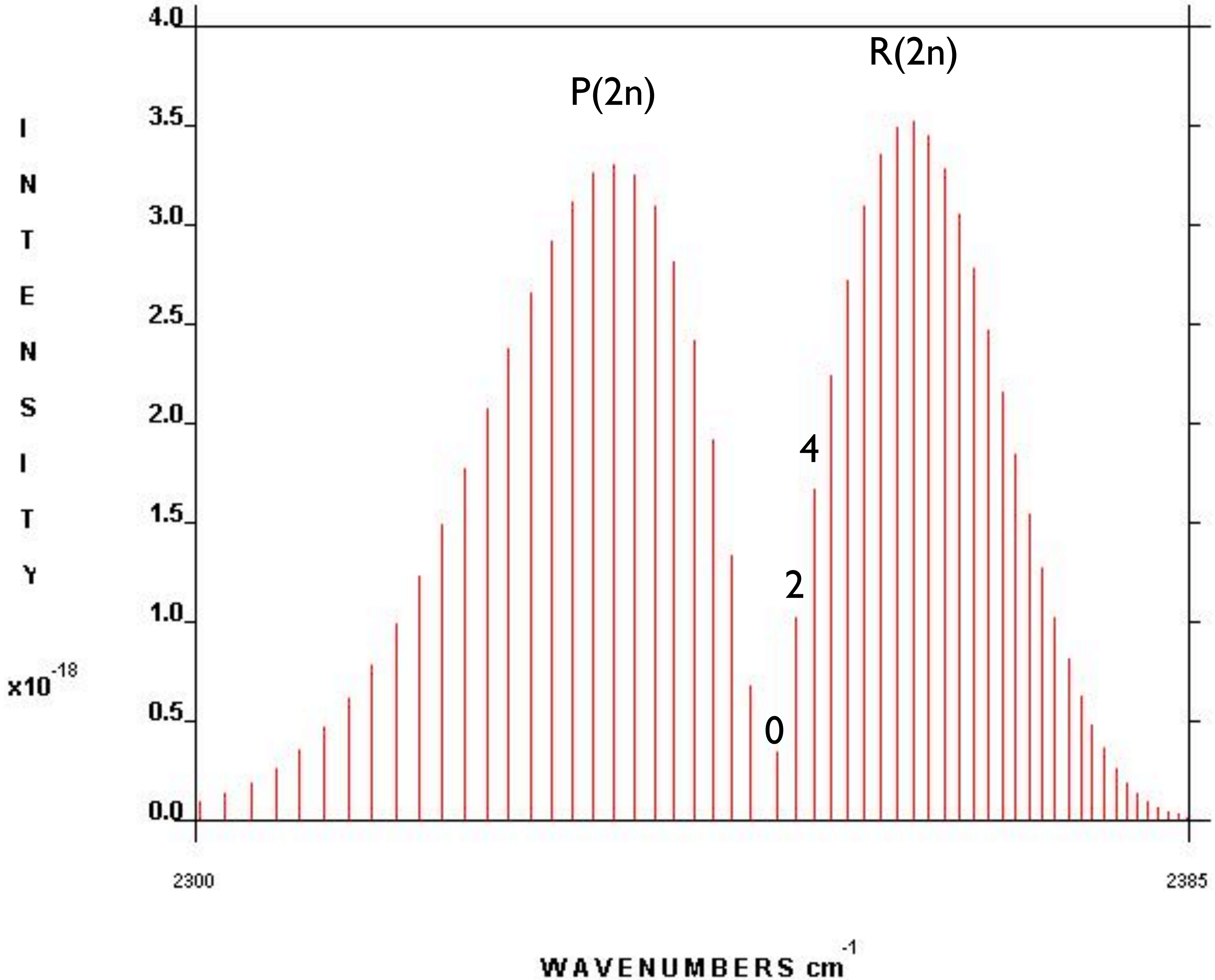
Forbidden internal states of molecules



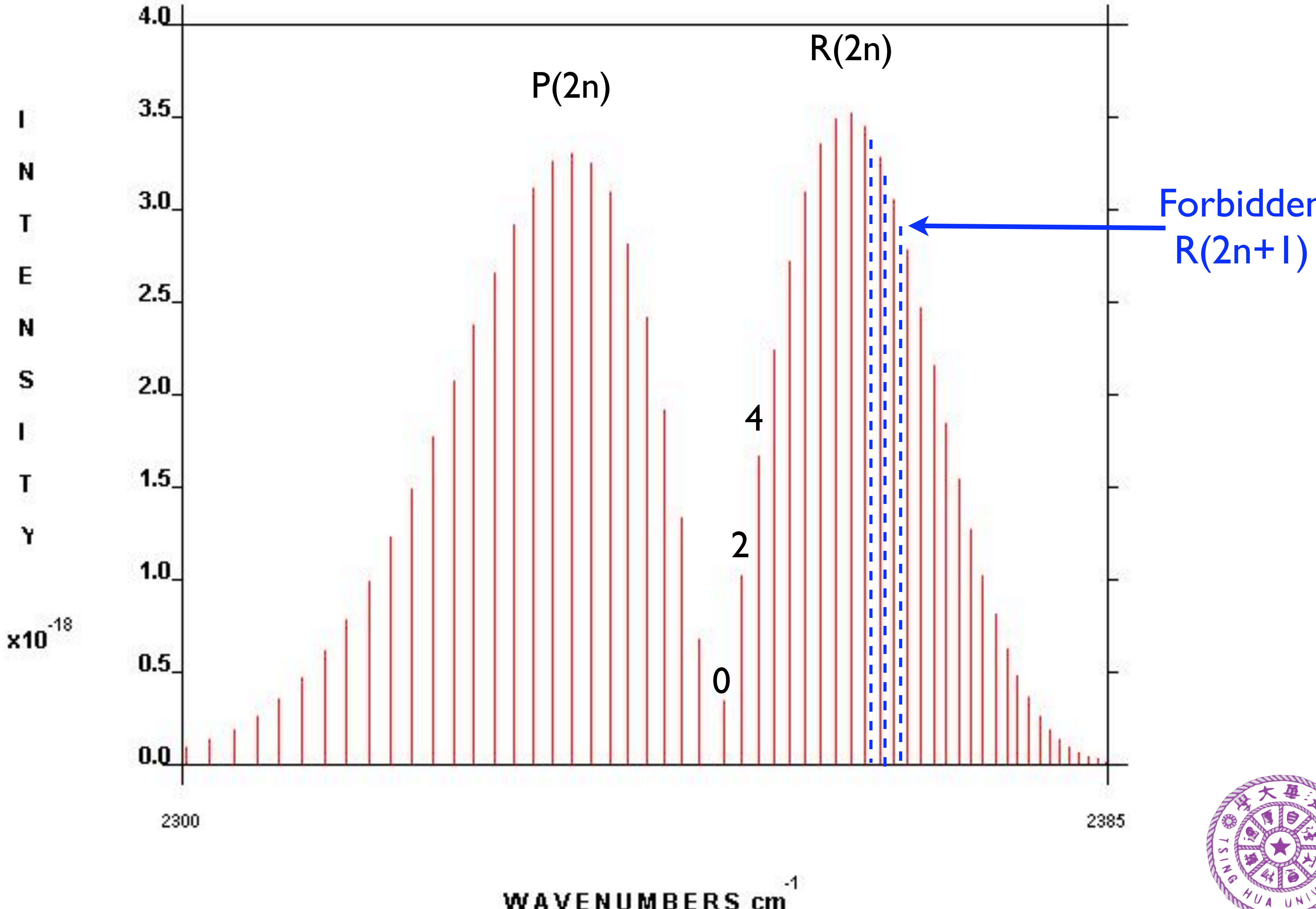
Forbidden internal states of molecules



The 4.3 μm spectrum of CO_2



The 4.3 μm spectrum of CO_2



Experimental Scheme

Searching for the very weak or even nonexistent J=(2n+1) transitions

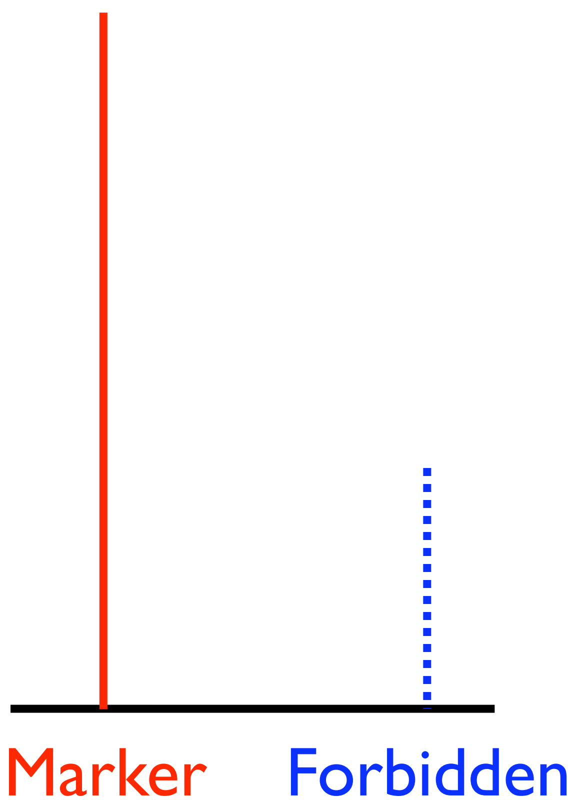
A neighboring marker line serves as both frequency and line intensity indicators

2 μm 00^00-12^01 R(25), G. Modugno, et al. (1998)

4.3 μm 00^00-00^01 R(25), D. Mazzotti, et al. (2001)

Q mutator

$$\rho_2 = (1 - \frac{1}{2}\beta^2)\rho_s + \frac{1}{2}\beta^2\rho_a$$



$$\frac{\beta^2}{2} = \frac{\mathbf{A}_f}{\mathbf{A}_m}\frac{\mathbf{S}_m}{\mathbf{S}_f}$$

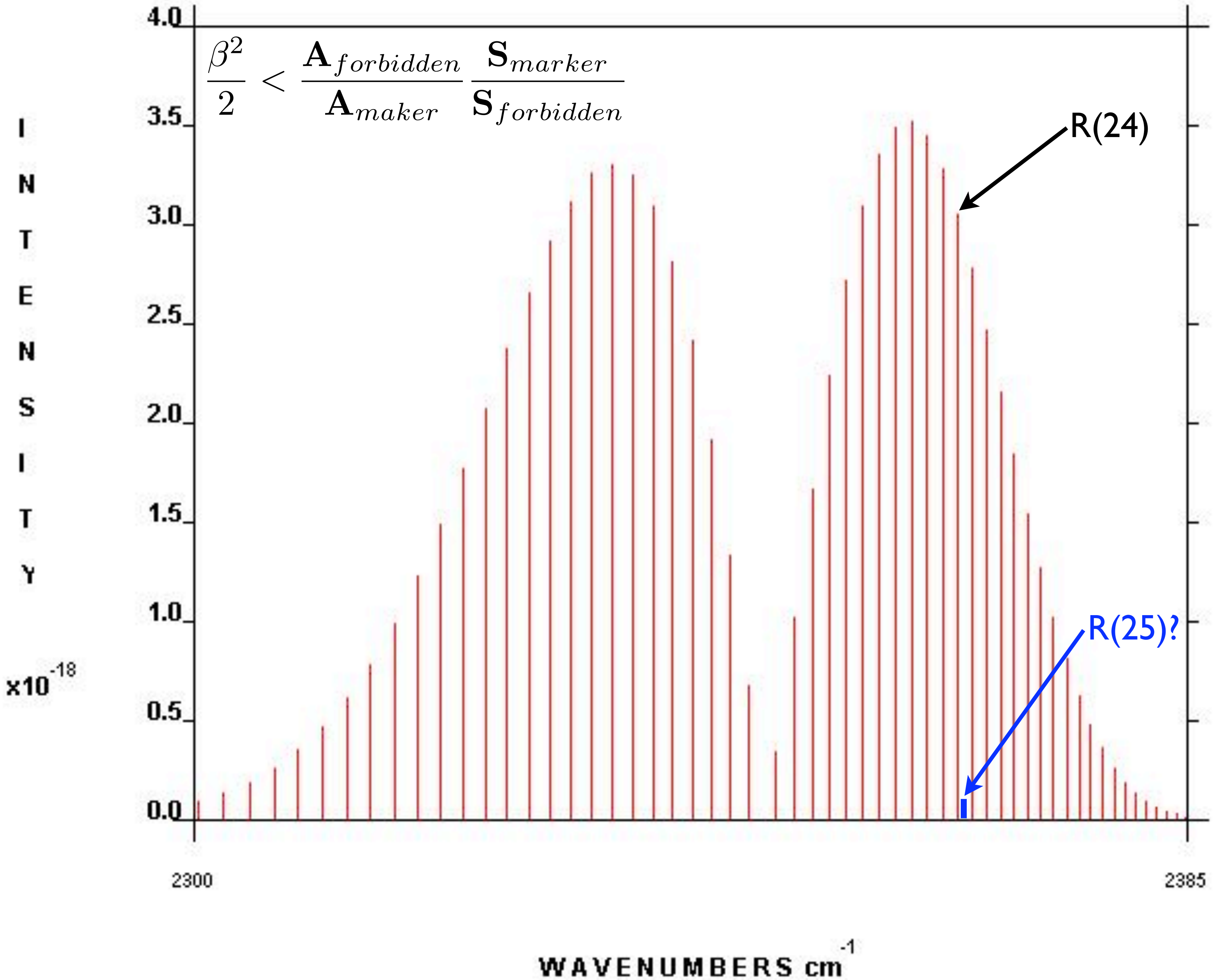
A: Signal Strength

S: Spectral Line Strength

$$\begin{aligned} S(J,T) &\propto (J+1)\exp\left(-\frac{h\nu_0 + E_r(J)}{k_B T}\right) \\ S(25,T) &\approx S(24,T)\frac{26}{25}\exp\left(-\frac{E_r(25) - E_r(24)}{k_B T}\right) \end{aligned}$$

$$\begin{aligned} \frac{\mathbf{A}_f}{\mathbf{A}_m} &= \frac{\mathbf{S}_f \times \frac{\beta^2}{2}}{\mathbf{S}_m} \\ \mathbf{A}_f &= \left(\frac{\beta^2}{2} \times \mathbf{S}_f\right) \times \frac{\mathbf{A}_m}{\mathbf{S}_m} \end{aligned}$$

The criterion for forbidden line

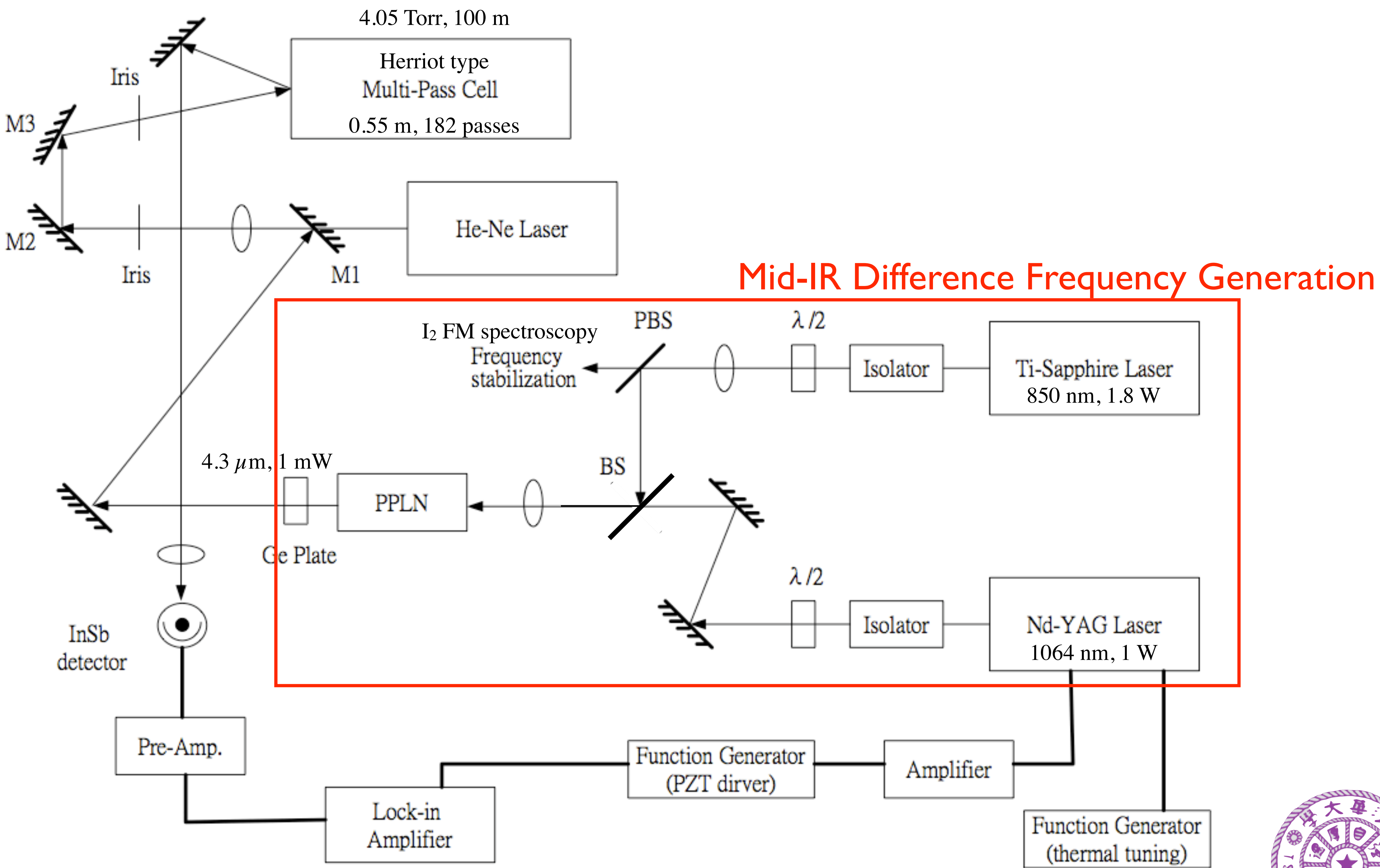


- Close to band peak
- No close-spaced strong line around it
- A well-separated weak line within frequency scanning range

Why CO_2

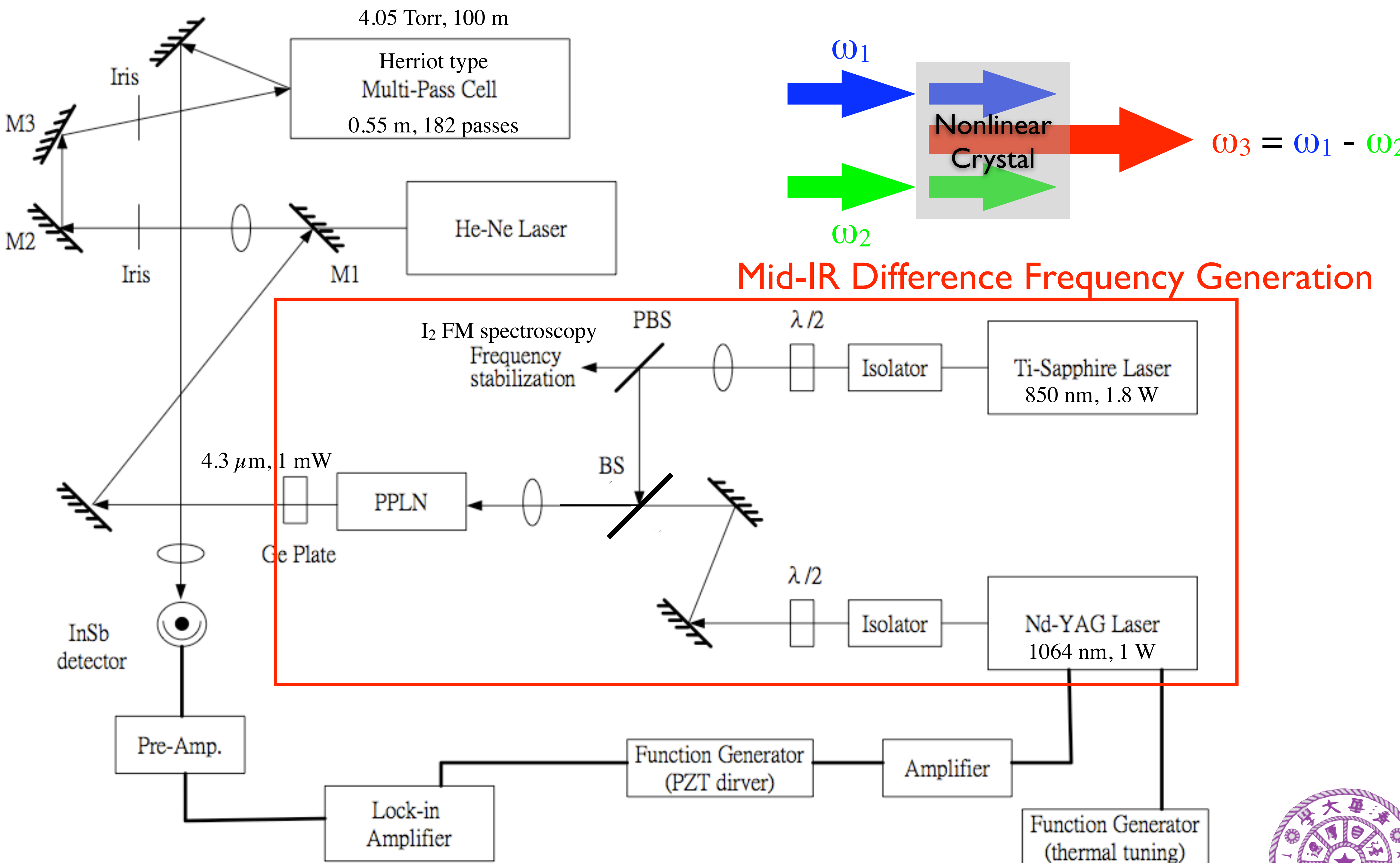
- A well-known molecule
- Very strong absorption around 4.3 μm
- Rich absorption lines
- High precision molecular constants available
  ★ good predictions of the forbidden line positions

Apparatus



4.05 Torr, 100 m
Herriot type
Multi-Pass Cell
0.55 m, 182 passes
Iris
M3
M2
Iris
M1
He-Ne Laser
Mid-IR Difference Frequency Generation
I2 FM spectroscopy
Frequency
stabilization
PBS
λ /2
Isolator
Ti-Sapphire Laser
850 nm, 1.8 W
4.3 μm, 1 mW
PPLN
BS
Ge Plate
λ /2
Isolator
Nd-YAG Laser
1064 nm, 1 W
InSb
detector
Pre-Amp.
Lock-in
Amplifier
Function Generator
(PZT dirver)
Amplifier
Function Generator
(thermal tuning)

Apparatus



ω_1
Nonlinear Crystal
ω_2
$\omega_3 = \omega_1 - \omega_2$
4.05 Torr, 100 m
Herriot type
Multi-Pass Cell
0.55 m, 182 passes
Iris
M3
M2
Iris
M1
He-Ne Laser
Mid-IR Difference Frequency Generation
I_2 FM spectroscopy
Frequency
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Isolator
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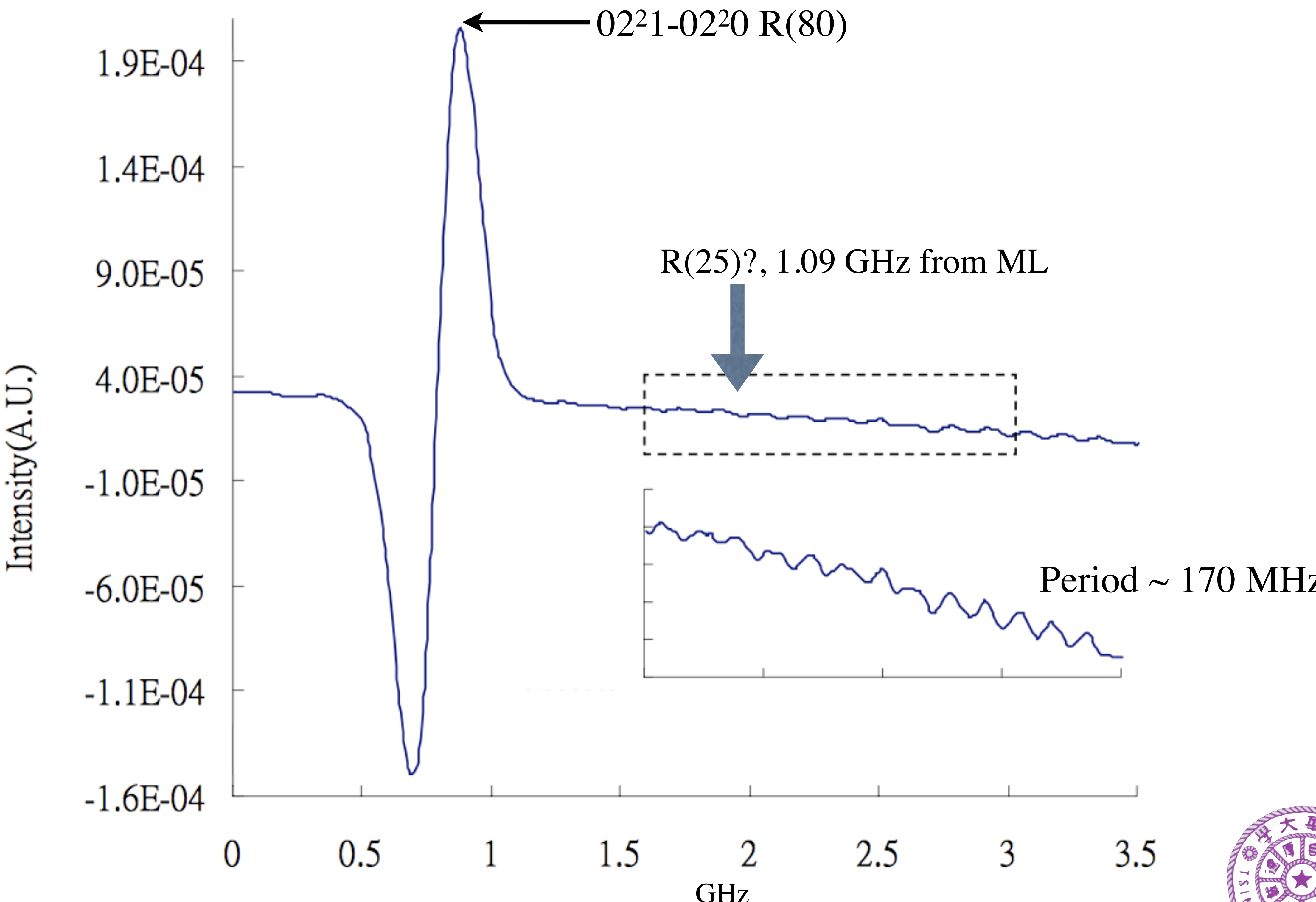
Spectrum

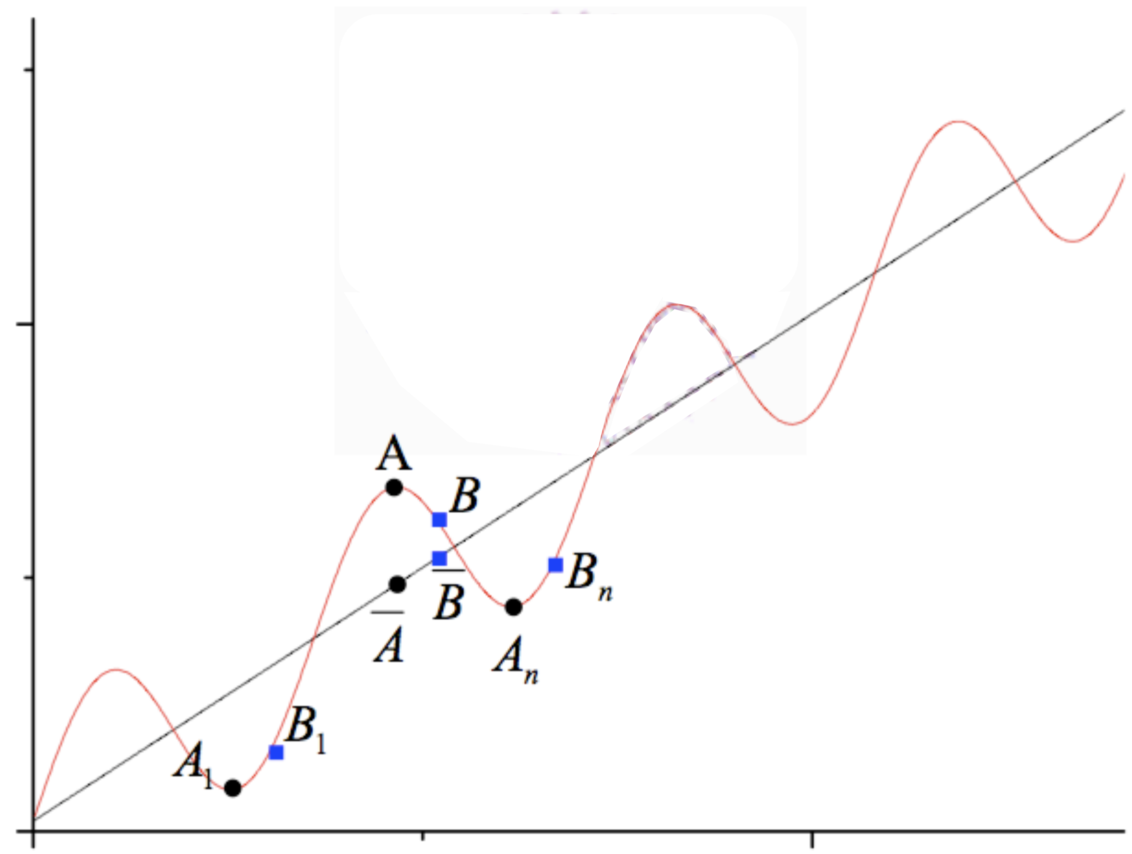


02^21-02^20 R(80)
R(25)?, 1.09 GHz from ML
Period ~ 170 MHz
1.9E-04
1.4E-04
9.0E-05
4.0E-05
-1.0E-05
-6.0E-05
-1.1E-04
-1.6E-04
Intensity(A.U.)
0
0.5
1
1.5
2
2.5
3
3.5
GHz

Fringe Reduction: Smooth

Smooth, Box Averaging, Moving Averaging

$$F(x_i) = \sum_{j=i-n/2}^{i+n/2} f(x_j)$$



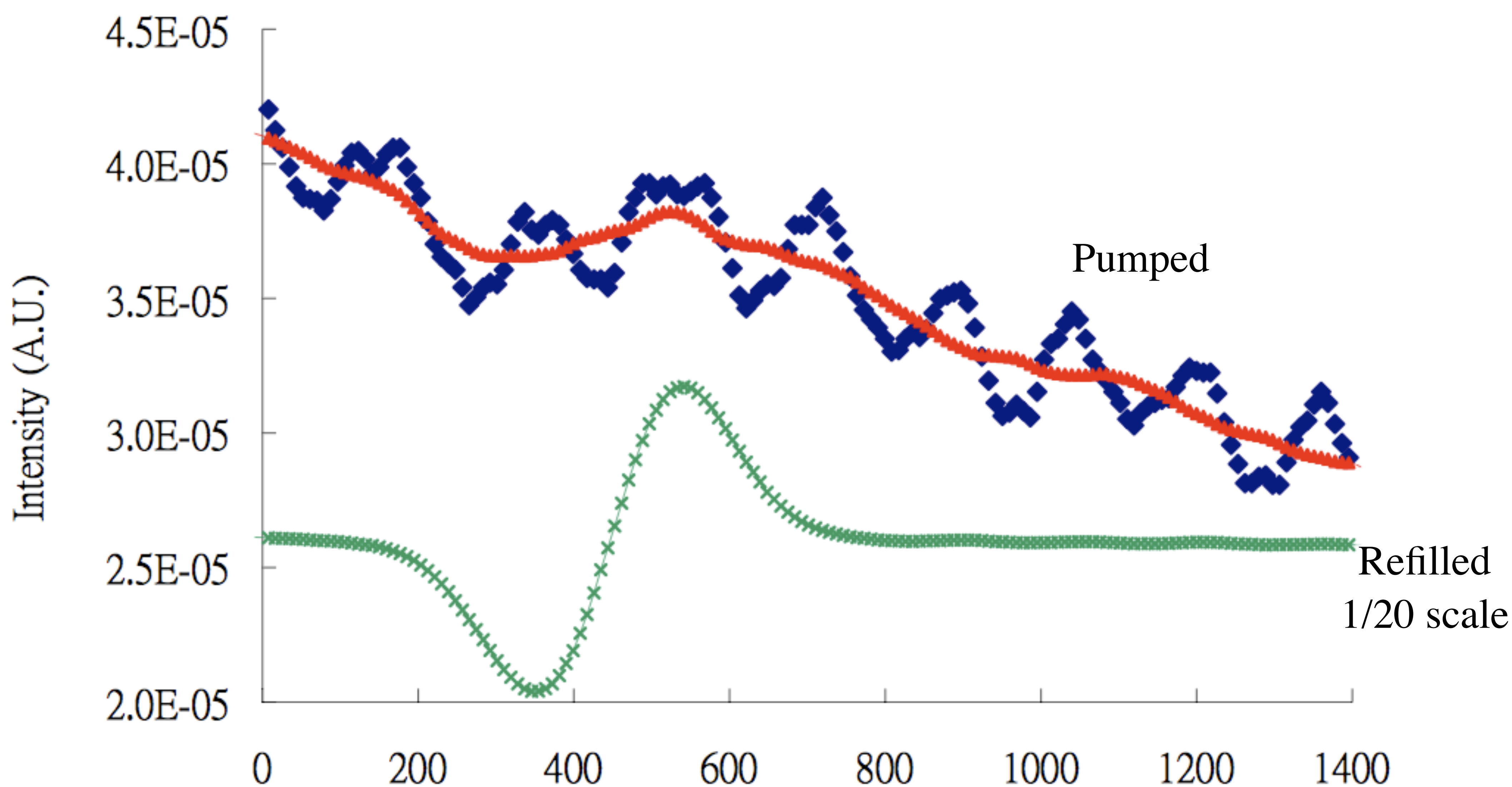
Advantages of smooth algorithm:

- Suppression of periodic pattern
- Noise cancellation

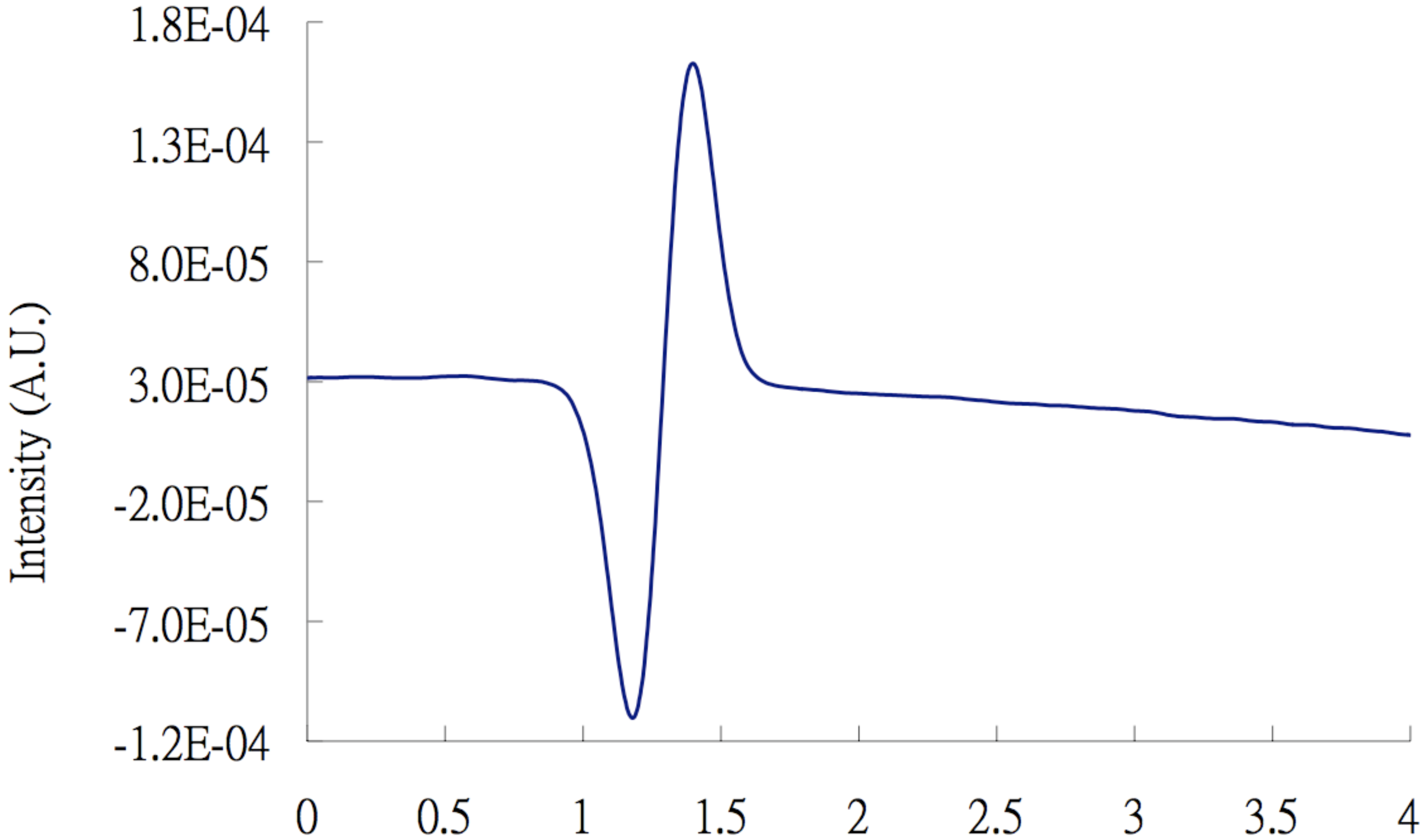
The smoothed derivative Gaussian

$$
\begin{aligned}
S(x) &= (ax+b) + A\frac{x-x_c}{W^2}\exp\left(-\frac{(x-x_c)^2}{2W^2}\right) \\
\bar{S}(x) &= \int_{x-L/2}^{x+L/2} S(x')dx' \\
&= (ax+b) - \frac{2A}{L}\exp\left(-\frac{(x-x_c)^2+L^2/4}{2W^2}\right) \times \sinh\frac{L(x-x_c)}{2W^2}
\end{aligned}
$$

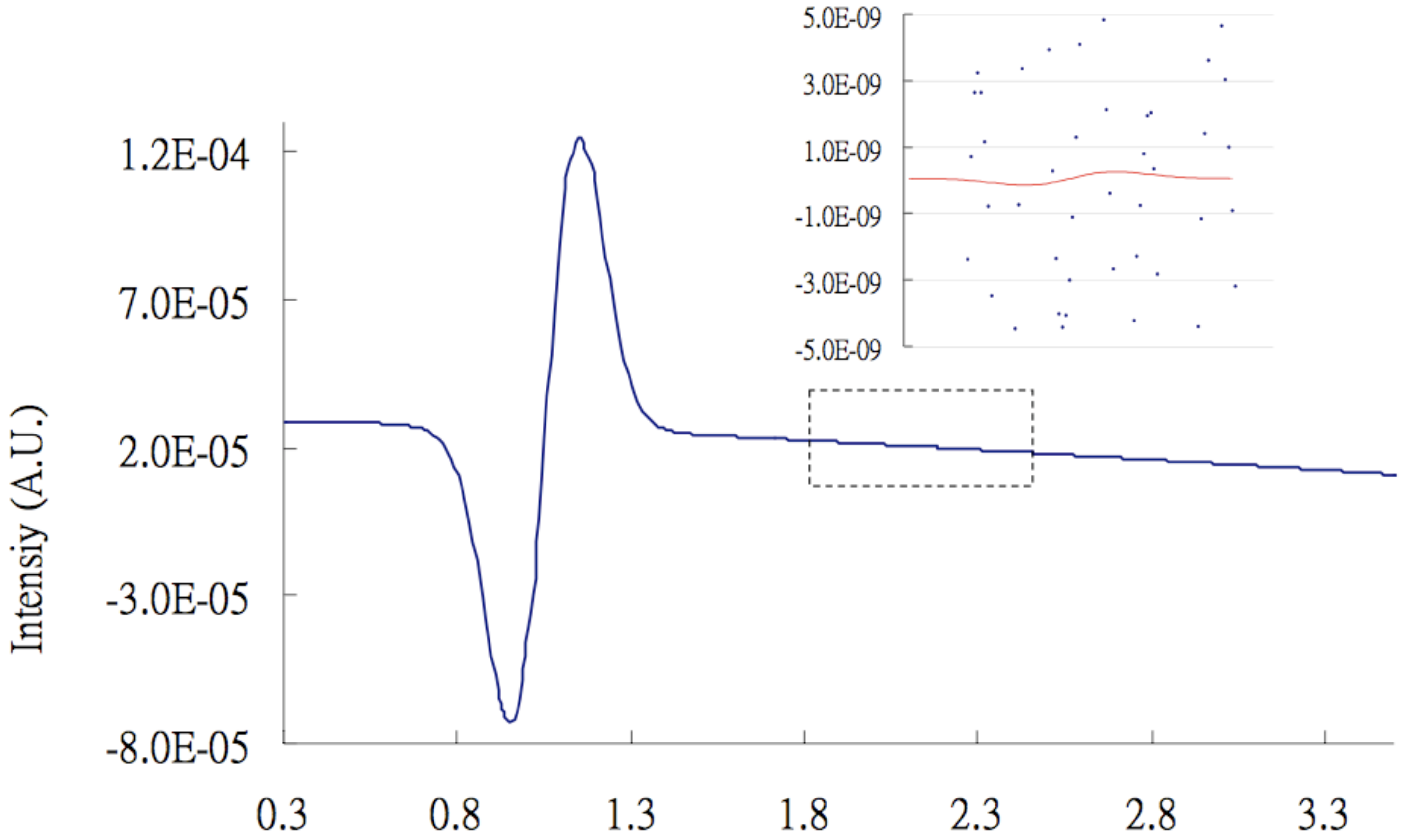
Effectiveness of smooth algorithm



Data Processing: Smoothing



Data Processing: Averaging 688 sets



Result

Estimating by RMS and Fitting, respectively

$$\frac{\beta^2}{2} < \frac{2.609 \times 10^{-9}}{0.974 \times 10^{-4}} \frac{2.29 \times 10^{-25}}{2.89 \times 10^{-18}} = 1.68 \times 10^{-12}$$

$$\frac{\beta^2}{2} < \frac{2.697 \times 10^{-8}}{2.250 \times 10^{-3}} \frac{2.29 \times 10^{-25}}{2.89 \times 10^{-18}} = 9.5 \times 10^{-14}$$

Future Works

- Suppressing the atmospheric absorption: only 20% optical power available for experiments.
- Locking DFG to optical frequency comb
  - Longer integration time
  - Smaller scanning steps
  - Smaller scanning range