

Qualifying Examination – Statistical Mechanics

Fall, 2015

1. (Fundamental concepts in statistical mechanics, 30 points)

Please EXPLAIN (not just describe or write equations/formula) the following terminologies in Statistical Mechanics.

- (a) Canonical Ensemble and Grand Canonical Ensemble
- (b) Fermi statistics and Fermi pressure
- (c) Gibbs paradox and its relationship with quantum statistics

2. (Entropy, 20 points)

Calculate the entropy of the following three-dimensional systems consisting of N atoms with atomic weight m and at temperature T ?

- (a) (5 pts) A crystal. At very low temperature, the interaction between these atoms can be neglected. Assume the ground state of each atom is doubly degenerate.
- (b) (5 pts) An ideal classical gas (free moving in space) with spin 2.
- (c) (10 pts) An ideal Fermi gas ($s=1/2$) with T much less than the Fermi temperature.

3. (Two-level system, 20 points)

Consider a system of N distinguishable particles, which have two energy levels, $E_0 = -\mu B$ and $E_1 = \mu B$, for each particles. Here μ is magnetic moment and B is magnetic field. The particles populate the energy levels according to the classical distribution law.

- (a) (10 pts) Calculate the average energy of such system at temperature T , and
- (b) (5 pts) the specific heat of the system.
- (c) (5 pts) Calculate the magnetic susceptibility.

4. (Ideal Fermi gas in 3-D, 30 points)

- (a) Considering an ideal Fermi gas of spin $S=5/2$ at zero temperature, calculate the total internal energy by summing the electron energy all the way up to the Fermi energy.
- (b) Calculate the Fermi Pressure, P .
- (c) Derive the Pauli magnetic susceptibility in the limit of zero magnetic field.